Chapter 10

10-1 From Eqs. (10-4) and (10-5)

\[ K_W - K_B = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} - \frac{4C + 2}{4C - 3} \]

Plot \( 100(K_W - K_B)/K_W \) vs. \( C \) for \( 4 \leq C \leq 12 \) obtaining

We see the maximum and minimum occur at \( C = 4 \) and 12 respectively where

Maximum = 1.36 % \( \text{Ans.} \), and Minimum = 0.743 % \( \text{Ans.} \).

10-2

\[ A = Sd^m \]

\[ \text{dim}(A_{\text{usc}}) = [\text{dim} (S) \text{ dim}(d^m)]_{\text{usc}} = \text{ksi} \cdot \text{in}^m \]

\[ \text{dim}(A_{\text{SI}}) = [\text{dim} (S) \text{ dim}(d^m)]_{\text{SI}} = \text{MPa} \cdot \text{mm}^m \]

\[ A_{\text{SI}} = \frac{\text{MPa}}{\text{ksi}} \cdot \frac{\text{in}^m}{\text{mm}^m} A_{\text{usc}} = 6.894757 (25.4)^m A_{\text{usc}} \bigg\rfloor 6.895 (25.4)^m A_{\text{usc}} \text{ Ans.} \]

For music wire, from Table 10-4:

\[ A_{\text{usc}} = 201 \text{ ksi} \cdot \text{in}^m, \quad m = 0.145; \quad \text{what is } A_{\text{SI}}? \]

\[ A_{\text{SI}} = 6.895(25.4)^{0.145} (201) = 2215 \text{ MPa} \cdot \text{mm}^m \quad \text{Ans.} \]

10-3 Given: Music wire, \( d = 2.5 \) mm, OD = 31 mm, plain ground ends, \( N_t = 14 \) coils.
(a) Table 10-1: \( N_a = N_t - 1 = 14 - 1 = 13 \) coils

\[ D = OD - d = 31 - 2.5 = 28.5 \text{ mm} \]

\[ C = D/d = 28.5/2.5 = 11.4 \]

Table 10-5: \( d = 2.5/25.4 = 0.098 \text{ in} \Rightarrow G = 81.0(10^3) \text{ MPa} \)

Eq. (10-9):

\[ k = \frac{d^4G}{8D^3N_a} = \frac{2.5^4(81)10^3}{8(28.5^3)13} = 1.314 \text{ N/mm} \quad \text{Ans.} \]

(b) Table 10-1: \( L_s = d N_t = 2.5(14) = 35 \text{ mm} \)

Table 10-4: \( m = 0.145, \quad A = 2211 \text{ MPa-mm}^m \)

Eq. (10-14):

\[ S_{st} = \frac{A}{d^m} = \frac{2211}{2.5^{0.145}} = 1936 \text{ MPa} \]

Table 10-6: \( S_{sy} = 0.45(1936) = 871.2 \text{ MPa} \)

Eq. (10-5):

\[ K_B = \frac{4C + 2}{4C - 3} = \frac{4(11.4) + 2}{4(11.4) - 3} = 1.117 \]

Eq. (10-7):

\[ F_s = \frac{\pi d^3 S_{st}}{8K_B D} = \frac{\pi(2.5^3)871.2}{8(1.117)28.5} = 167.9 \text{ N} \quad \text{Ans.} \]

(c)

\[ L_0 = \frac{F_s}{k} + L_s = \frac{167.9}{1.314} + 35 = 162.8 \text{ mm} \quad \text{Ans.} \]

(d) \( (L_0)_{cr} = \frac{2.63(28.5)}{0.5} = 149.9 \text{ mm} \). Spring needs to be supported. \text{Ans.} 

10-4 Given: Design load, \( F_1 = 130 \text{ N} \).

Referring to Prob. 10-3 solution, \( C = 11.4, N_a = 13 \) coils, \( S_{sy} = 871.2 \text{ MPa}, F_s = 167.9 \text{ N}, L_0 = 162.8 \text{ mm and } (L_0)_{cr} = 149.9 \text{ mm} \).

Eq. (10-18): \( 4 \leq C \leq 12 \quad C = 11.4 \quad \text{O.K.} \)

Eq. (10-19): \( 3 \leq N_a \leq 15 \quad N_a = 13 \quad \text{O.K.} \)

Eq. (10-17):

\[ \xi = \frac{F_s}{F_1} - 1 = \frac{167.9}{130} - 1 = 0.29 \]
Eq. (10-20): \( \xi \geq 0.15, \quad \xi = 0.29 \quad O.K. \)
From Eq. (10-7) for static service
\[
\tau_s = K_B \left( \frac{8F_D}{\pi d^3} \right) = 1.117 \left( \frac{8(130)(28.5)}{\pi(2.5)^3} \right) = 674 \text{ MPa}
\]
\[
n = \frac{S_{sy}}{\tau_s} = \frac{871.2}{674} = 1.29
\]
Eq. (10-21): \( n_s \geq 1.2, \quad n = 1.29 \quad O.K. \)
\[
\tau_s = \frac{167.9}{130} = 674 \left( \frac{167.9}{130} \right) = 870.5 \text{ MPa}
\]
\[
S_{sy} / \tau_s = 871.2 / 870.5 \geq 1
\]
\( S_{sy} / \tau_s \geq (n_s)_d \): Not solid-safe (but was the basis of the design). \( Not \ O.K. \)

\( L_0 \leq (L_0)_c \): 162.8 \( \geq \) 149.9 \( Not \ O.K. \)

Design is unsatisfactory. Operate over a rod? \( Ans. \)

**10-5**

**Given:** Oil-tempered wire, \( d = 0.2 \text{ in}, \quad D = 2 \text{ in}, \quad N_t = 12 \text{ coils}, \quad L_0 = 5 \text{ in}, \quad \text{squared ends}.**

- **(a)** Table 10-1: \( L_s = d (N_t + 1) = 0.2(12 + 1) = 2.6 \text{ in} \quad Ans. \)

- **(b)** Table 10-1: \( N_u = N_t - 2 = 12 - 2 = 10 \text{ coils} \)
  
  Table 10-5: \( G = 11.2 \text{ Mpsi} \)

  **Eq. (10-9):**
  
  \[
  k = \frac{d^4G}{8D^3N_u} = \frac{0.2^4(11.2)10^6}{8(2^3)10} = 28 \text{ lbf/in}
  \]
  
  \[
  F_s = k y_s = k (L_0 - L_s) = 28(5 - 2.6) = 67.2 \text{ lbf} \quad Ans.
  \]

- **(c)** Eq. (10-1): \( C = D/d = 2/0.2 = 10 \)

  **Eq. (10-5):**
  
  \[
  K_B = \frac{4C + 2}{4C - 3} = \frac{4(10) + 2}{4(10) - 3} = 1.135
  \]

  **Eq. (10-7):**
  
  \[
  \tau_s = K_B \left( \frac{8F_D}{\pi d^3} \right) = 1.135 \left( \frac{8(67.2)2}{\pi(0.2^3)} \right) = 48.56(10^3) \text{ psi}
  \]

  **Table 10-4:** \( m = 0.187, \quad A = 147 \text{ kpsi-in}^m \)
Eq. (10-14):  \[ S_{ut} = \frac{A}{d^m} = \frac{147}{0.2^{0.187}} = 198.6 \text{ kpsi} \]

Table 10-6:  \[ S_{sy} = 0.50 \ S_{ut} = 0.50(198.6) = 99.3 \text{ kpsi} \]

\[
\frac{n_s}{\tau_s} = \frac{99.3}{48.56} = 2.04 \quad \text{Ans.}
\]

---

10-6  Given: Oil-tempered wire, \( d = 4 \text{ mm} \), \( C = 10 \), plain ends, \( L_0 = 80 \text{ mm} \), and at \( F = 50 \text{ N} \), \( y = 15 \text{ mm} \).

(a)  \[ k = \frac{F}{y} = \frac{50}{15} = 3.333 \text{ N/mm} \quad \text{Ans.} \]

(b)  \[ D = Cd = 10(4) = 40 \text{ mm} \]

\[ \text{OD} = D + d = 40 + 4 = 44 \text{ mm} \quad \text{Ans.} \]

(c) From Table 10-5, \( G = 77.2 \text{ GPa} \)

Eq. (10-9):  \[ N_a = \frac{d^4G}{8kD^3} = \frac{4^4(77.2)10^3}{8(3.333)40^3} = 11.6 \text{ coils} \]

Table 10-1:  \[ N_t = N_a = 11.6 \text{ coils} \quad \text{Ans.} \]

(d) Table 10-1:  \[ L_s = d \ (N_t + 1) = 4(11.6 + 1) = 50.4 \text{ mm} \quad \text{Ans.} \]

(e) Table 10-4:  \[ m = 0.187, \ A = 1855 \text{ MPa-mm}^m \]

Eq. (10-14):  \[ S_{ut} = \frac{A}{d^m} = \frac{1855}{4^{0.187}} = 1431 \text{ MPa} \]

Table 10-6:  \[ S_{sy} = 0.50 \ S_{ut} = 0.50(1431) = 715.5 \text{ MPa} \]

\[ y_s = L_0 - L_s = 80 - 50.4 = 29.6 \text{ mm} \]

\[ F_s = ky_s = 3.333(29.6) = 98.66 \text{ N} \]

Eq. (10-5):  \[ K_B = \frac{4C + 2}{4C - 3} = \frac{4(10) + 2}{4(10) - 3} = 1.135 \]

Eq. (10-7):  \[ \tau_s = K_B \frac{8FD}{\pi d^3} = 1.135 \frac{8(98.66)40}{\pi (4^3)} = 178.2 \text{ MPa} \]
### 10-7 Static service spring with: HD steel wire, \( \text{d} = 0.080 \text{ in}, \text{OD} = 0.880 \text{ in}, N_t = 8 \) coils, plain and ground ends.

**Preliminaries**

<table>
<thead>
<tr>
<th>Table 10-5: ( A = 140 \text{ kpsi} \cdot \text{in}^m, \ m = 0.190 )</th>
</tr>
</thead>
</table>

**Eq. (10-14):** \[ S_{\text{us}} = \frac{A}{d^m} = \frac{140}{0.080^{0.190}} = 226.2 \text{ kpsi} \]

<table>
<thead>
<tr>
<th>Table 10-6: ( S_{\text{us}} = 0.45(226.2) = 101.8 \text{ kpsi} )</th>
</tr>
</thead>
</table>

Then,

**Eq. (10-1):** \( D = \text{OD} - \text{d} = 0.880 - 0.080 = 0.8 \text{ in} \)

**Eq. (10-5):** \( K_B = \frac{4C + 2}{4C - 3} = \frac{4(10) + 2}{4(10) - 3} = 1.135 \)

| Table 10-1: \( N_a = N_t - 1 = 8 - 1 = 7 \) coils |

**Eq. (10-7):** For solid-safe, \( n_s = 1.2 \)

\[
F_s = \frac{\pi d^3 S_{\text{us}} / n_s}{8K_B D} = \frac{\pi (0.08^3)[101.8(10^3) / 1.2]}{8(1.135)(0.8)} = 18.78 \text{ lbf}
\]

**Eq. (10-9):** \[ k = \frac{d^4G}{8D^3N_a} = \frac{0.08^4(115)10^6}{8(0.8^3)7} = 16.43 \text{ lbf/in} \]

**Eq. (10-18):** \[ 4 \leq C \leq 12 \quad C = 10 \quad \text{O.K.} \]

---

### 10-8 Given: Design load, \( F_1 = 16.5 \text{ lbf}. \)

Referring to Prob. 10-7 solution, \( C = 10, N_t = 7 \) coils, \( S_{\text{us}} = 101.8 \text{ kpsi}, F_s = 18.78 \text{ lbf}, \ y_s = 1.14 \text{ in}, L_0 = 1.78 \text{ in}, \) and \( (L_0)_{cr} = 4.21 \text{ in}. \)

**Eq. (10-18):** \[ 4 \leq C \leq 12 \quad C = 10 \quad \text{O.K.} \]
Eq. (10-19): \[3 \leq N_a \leq 15\quad N_a = 7 \quad O.K.

Eq. (10-17): \[\xi = \frac{F}{F_1} - 1 = \frac{18.78}{16.5} - 1 = 0.14\]

Eq. (10-20): \[\xi \geq 0.15,\quad \xi = 0.14\quad \text{not O.K., but probably acceptable.}\]

From Eq. (10-7) for static service
\[\tau_t = \frac{8F_dD}{\pi d^3} = 1.135\frac{8(16.5)(0.8)}{\pi(0.080)^3} = 74.5 \left(10^3\right) \text{ psi} = 74.5 \text{ kpsi}\]
\[n = \frac{S_n}{\tau_t} = \frac{101.8}{74.5} = 1.37\]

Eq. (10-21): \[n_s \geq 1.2,\quad n = 1.37\quad O.K.
\[\tau_s = \frac{18.78}{16.5} = 74.5 \left(\frac{18.78}{16.5}\right) = 84.8 \text{ kpsi}\]
\[n_s = \frac{S_{sy}}{\tau_s} = \frac{101.8}{84.8} = 1.2\]

Eq. (10-21): \[n_s \geq 1.2,\quad n_s = 1.2\quad \text{It is solid-safe (basis of design).}\quad O.K.

Eq. (10-13) and Table 10-2: \[L_0 \leq (L_0)_{cr} \quad 1.78 \text{ in} \leq 4.21 \text{ in}\quad O.K.

---

10-9 Given: A228 music wire, squared and ground ends, \(d = 0.007\) in, OD = 0.038 in, \(L_0 = 0.58\) in, \(N_t = 38\) coils.

\[D = \text{OD} - d = 0.038 - 0.007 = 0.031\ \text{in}\]

Eq. (10-1): \[C = \frac{D}{d} = \frac{0.031}{0.007} = 4.429\]

Eq. (10-5): \[K_b = \frac{4C + 2}{4C - 3} = \frac{4(4.429) + 2}{4(4.429) - 3} = 1.340\]

Table 10-1: \(N_a = N_t - 2 = 38 - 2 = 36\) coils (high)

Table 10-5: \(G = 12.0\ \text{Mpsi}\)

Eq. (10-9): \[k = \frac{d^4G}{8D^3N_a} = \frac{0.007^4(12.0)10^6}{8(0.031^3)36} = 3.358\ \text{lbf/in}\]

Table 10-1: \(L_s = dN_t = 0.007(38) = 0.266\) in
\(y_s = L_0 - L_s = 0.58 - 0.266 = 0.314\) in
\(F_s = k y_s = 3.358(0.314) = 1.054\ \text{lbf}\)

Eq. (10-7): \[\tau_s = \frac{8F_dD}{\pi d^3} = 1.340\frac{8(1.054)(0.031)}{\pi(0.007^3)} = 325.1 \left(10^3\right) \text{ psi}\quad (1)

Table 10-4: \(A = 201\ \text{ksi-in}^m,\quad m = 0.145\)
Eq. (10-14): \[ S' = \frac{A}{d^m} = \frac{201}{0.007^{0.145}} = 412.7 \text{ kpsi} \]

Table 10-6: \[ S_{sy} = 0.45 \text{ } S' = 0.45(412.7) = 185.7 \text{ kpsi} \]

\[ \tau_s > S_{sy}, \text{ that is, } 325.1 > 185.7 \text{ kpsi, the spring is not solid-safe. Return to Eq. (1) with } \]

\[ F_s = ky_s \text{ and } \tau_s = S_{sy}/n_s \text{, and solve for } y_s \text{, giving } \]

\[ y_s = \left( \frac{S_{sy} / n_s}{\pi d^3} \right) \frac{8K_p kD}{8} \left( \frac{185.7 (10^3) / 1.2}{\pi (0.007^3)} \right) = 0.149 \text{ in} \]

The free length should be wound to

\[ L_0 = L_s + y_s = 0.266 + 0.149 = 0.415 \text{ in} \quad \text{Ans.} \]

This only addresses the solid-safe criteria. There are additional problems.

**10-10** Given: B159 phosphor-bronze, squared and ground. ends, \( d = 0.014 \text{ in} \), OD = 0.128 in, \( L_0 = 0.50 \text{ in} \), \( N_t = 16 \text{ coils} \).

\[ D = \text{OD} - d = 0.128 - 0.014 = 0.114 \text{ in} \]

Eq. (10-1): \[ C = D/d = 0.114/0.014 = 8.143 \]

Eq. (10-5): \[ K_B = \frac{4C + 2}{4C - 3} = \frac{4(8.143) + 2}{4(8.143) - 3} = 1.169 \]

Table 10-1: \( N_a = N_t - 2 = 16 - 2 = 14 \text{ coils} \)

Table 10-5: \( G = 6 \text{ Mpsi} \)

Eq. (10-9): \[ k = \frac{d^4 G}{8 D^3 N_a} = \frac{0.014^4 (6) 10^6}{8(0.114^3) 14} = 1.389 \text{ lbf/in} \]

Table 10-1: \[ L_s = dN_t = 0.014(16) = 0.224 \text{ in} \]

\[ y_s = L_s - L_s = 0.50 - 0.224 = 0.276 \text{ in} \]

\[ F_s = ky_s = 1.389(0.276) = 0.3834 \text{ lbf} \]

Eq. (10-7): \[ \tau_s = K_B \frac{8 F_s D}{\pi d^3} = 1.169 \frac{8(0.3834)(0.141)}{\pi (0.014^3)} = 47.42(10^3) \text{ psi} \quad (1) \]

Table 10-4: \[ A = 145 \text{ kpsi-in}^m, \quad m = 0 \]

Eq. (10-14): \[ S' = \frac{A}{d^m} = \frac{145}{0.014^0} = 145 \text{ kpsi} \]

Table 10-6: \[ S_{sy} = 0.35 S' = 0.35(145) = 47.25 \text{ kpsi} \]

\[ \tau_s > S_{sy}, \text{ that is, } 47.42 > 47.25 \text{ kpsi, the spring is not solid-safe. Return to Eq. (1) with } \]

\[ F_s = ky_s \text{ and } \tau_s = S_{sy}/n_s \text{, and solve for } y_s \text{, giving } \]

\[ y_s = \left( \frac{S_{sy} / n_s}{\pi d^3} \right) \frac{8K_p kD}{8} \left( \frac{47.25(10^3) / 1.2}{\pi (0.014^3)} \right) = 0.229 \text{ in} \]

The free length should be wound to
\[ L_0 = L_s + y_s = 0.224 + 0.229 = 0.453 \text{ in} \quad \text{Ans.} \]

**10-11** Given: A313 stainless steel, squared and ground ends, \( d = 0.050 \) in, OD = 0.250 in, \( L_0 = 0.68 \) in, \( N_t = 11.2 \) coils.

\[
D = \text{OD} - d = 0.250 - 0.050 = 0.200 \text{ in}
\]

Eq. (10-1): \[
C = \frac{D}{d} = \frac{0.200}{0.050} = 4
\]

Eq. (10-5): \[
K_B = \frac{4C + 2}{4C - 3} = \frac{4(4) + 2}{4(4) - 3} = 1.385
\]

Table 10-1: \( N_a = N_t - 2 = 11.2 - 2 = 9.2 \) coils

Table 10-5: \( G = 10 \text{ Mpsi} \)

Eq. (10-9): \[
k = \frac{d^4G}{8D^3N_a} = \frac{0.050^4(10)10^6}{8(0.2^3)9.2} = 106.1 \text{ lbf/in}
\]

Table 10-1: \( L_a = dN_t = 0.050(11.2) = 0.56 \) in

\( y_s = L_0 - L_a = 0.68 - 0.56 = 0.12 \) in

\( F_s = k y_s = 106.1(0.12) = 12.73 \text{ lbf} \)

Eq. (10-7): \[
\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.385 \frac{8(12.73)0.2}{\pi(0.050^3)} = 71.8(10^3) \text{ psi}
\]

Table 10-4: \( A = 169 \text{ kpsi-in}^m, m = 0.146 \)

Eq. (10-14): \[
S_{ut} = \frac{A}{d^m} = \frac{169}{0.050^{0.146}} = 261.7 \text{ kpsi}
\]

Table 10-6: \( S_{sy} = 0.35 S_{ut} = 0.35(261.7) = 91.6 \text{ kpsi} \)

\[
n_s = \frac{S_{sy}}{\tau_s} = \frac{91.6}{71.8} = 1.28 \quad \text{Spring is solid-safe} \quad (n_s > 1.2) \quad \text{Ans.}
\]

**10-12** Given: A227 hard-drawn wire, squared and ground ends, \( d = 0.148 \) in, OD = 2.12 in, \( L_0 = 2.5 \) in, \( N_t = 5.75 \) coils.

\[
D = \text{OD} - d = 2.12 - 0.148 = 1.972 \text{ in}
\]

Eq. (10-1): \[
C = \frac{D}{d} = \frac{1.972}{0.148} = 13.32 \quad \text{(high)}
\]

Eq. (10-5): \[
K_B = \frac{4C + 2}{4C - 3} = \frac{4(13.32) + 2}{4(13.32) - 3} = 1.099
\]

Table 10-1: \( N_a = N_t - 2 = 5.75 - 2 = 3.75 \) coils

Table 10-5: \( G = 11.4 \text{ Mpsi} \)

Eq. (10-9): \[
k = \frac{d^4G}{8D^3N_a} = \frac{0.148^4(11.4)10^6}{8(1.972^3)3.75} = 23.77 \text{ lbf/in}
\]

Table 10-1: \( L_a = dN_t = 0.148(5.75) = 0.851 \text{ in} \)

\( y_s = L_0 - L_a = 2.5 - 0.851 = 1.649 \text{ in} \)

\( F_s = k y_s = 23.77(1.649) = 39.20 \text{ lbf} \)
Eq. (10-7): \[ \tau_s = K_b \frac{8FD}{\pi d^3} = 1.099 \frac{8(39.20)1.972}{\pi (0.148^3)} = 66.7\left(10^3\right) \text{psi} \]

Table 10-4: \( A = 140 \text{ kpsi-in}^m, m = 0.190 \)

Eq. (10-14): \( S_{ut} = \frac{A}{d^m} = \frac{140}{0.148^{0.190}} = 201.3 \text{ kpsi} \)

Table 10-6: \( S_{sy} = 0.35 S_{ut} = 0.45(201.3) = 90.6 \text{ kpsi} \)

\[ n_s = \frac{S_{sy}}{\tau_s} = \frac{90.6}{66.7} = 1.36 \]

Spring is solid-safe \((n_s > 1.2)\) \( \text{Ans.} \)

---

**10-13** Given: A229 OQ&T steel, squared and ground ends, \( d = 0.138 \text{ in}, \text{OD} = 0.92 \text{ in}, \)
\( L_0 = 2.86 \text{ in}, N_t = 12 \text{ coils}. \)

\[ D = \text{OD} - d = 0.92 - 0.138 = 0.782 \text{ in} \]

Eq. (10-1): \[ C = D/d = 0.782/0.138 = 5.667 \]

Eq. (10-5): \[ K_b = \frac{4C + 2}{4C - 3} = \frac{4(5.667) + 2}{4(5.667) - 3} = 1.254 \]

Table 10-1: \( N_a = N_t - 2 = 12 - 2 = 10 \text{ coils} \)

A229 OQ&T steel is not given in Table 10-5. From Table A-5, for carbon steels, \( G = 11.5 \text{ Mpsi}. \)

Eq. (10-9): \[ k = D^4G \frac{1.38^4 (11.5)10^6}{8D^3N_a} = \frac{0.138^4 (11.5)10^6}{8(0.782^3)10} = 109.0 \text{ lbf/in} \]

Table 10-1: \( L_s = dN_a = 0.138(12) = 1.656 \text{ in} \)
\( y_s = L_0 - L_s = 2.86 - 1.656 = 1.204 \text{ in} \)
\( F_s = ky_s = 109.0(1.204) = 131.2 \text{ lbf} \)

Eq. (10-7): \[ \tau_s = K_b \frac{8FD}{\pi d^3} = 1.254 \frac{8(131.2)0.782}{\pi (0.138^3)} = 124.7\left(10^3\right) \text{ psi} \quad (1) \]

Table 10-4: \( A = 147 \text{ kpsi-in}^m, m = 0.187 \)

Eq. (10-14): \( S_{ut} = \frac{A}{d^m} = \frac{147}{0.138^{0.187}} = 212.9 \text{ kpsi} \)

Table 10-6: \( S_{sy} = 0.50 S_{ut} = 0.50(212.9) = 106.5 \text{ kpsi} \)

\( \tau_s > S_{sy}, \) that is, 124.7 > 106.5 kpsi, the spring is not solid-safe. Return to Eq. (1) with \( F_s = ky_s \) and \( \tau_s = S_{sy}/n_s, \) and solve for \( y_s, \) giving

\[ y_s = \frac{\left(S_{sy}/n_s\right) \pi d^3}{8K_b^2kD} = \left[ \frac{106.5\left(10^3\right)/1.2}{8(1.254)109.0(0.782)} \right] = 0.857 \text{ in} \]

The free length should be wound to
\[ L_0 = L_s + y_s = 1.656 + 0.857 = 2.51 \text{ in} \quad \text{Ans.} \]

**10-14** Given: A232 chrome-vanadium steel, squared and ground ends, \( d = 0.185 \text{ in}, \ OD = 2.75 \text{ in}, \ L_0 = 7.5 \text{ in}, \ N_I = 8 \text{ coils}.\)

\[
D = OD - d = 2.75 - 0.185 = 2.565 \text{ in}
\]

Eq. (10-1): \[ C = \frac{4C + 2}{4C - 3} = \frac{4(13.86) + 2}{4(13.86) - 3} = 1.095 \]

Table 10-1: \[ N_a = N_I - 2 = 8 - 2 = 6 \text{ coils} \]

Table 10-5: \[ G = 11.2 \text{ Mpsi.} \]

Eq. (10-9): \[ F_s = k \frac{G}{\pi d^3} \left[ \frac{1.095}{11.2} \right] = 97.5 \text{ lbf} \]

Table 10-4: \[ A = 169 \text{ kpsi-in}^m, \ m = 0.168 \]

Eq. (10-14): \[ S_{uy} = \frac{A}{d^m} = \frac{169}{0.185^{0.168}} = 224.4 \text{ kpsi} \]

Table 10-6: \[ S_{uy} = 0.50 \text{ S}_{uy} = 0.50(224.4) = 112.2 \text{ kpsi} \]

\[ n_s = \frac{S_{uy}}{S_{uy}} = \frac{112.2}{110.1} = 1.02 \quad \text{Spring is not solid-safe} \ (n_s < 1.2) \]

Return to Eq. (1) with \( F = ky_s \) and \( \tau = \frac{S_{uy}}{n_s} \), and solve for \( y_s \), giving

\[ y_s = \frac{\left( S_{uy} / n_s \right) \pi d^3}{8K_B k D} = \frac{112.2(10^3)/1.2}{8(0.1095)16.20(2.565)} = 5.109 \text{ in} \]

The free length should be wound to

\[ L_0 = L_s + y_s = 1.48 + 5.109 = 6.59 \text{ in} \quad \text{Ans.} \]

\[ L_0 = L_s + y_s = 1.656 + 0.857 = 2.51 \text{ in} \quad \text{Ans.} \]

**10-15** Given: A313 stainless steel, squared and ground ends, \( d = 0.25 \text{ mm}, \ OD = 0.95 \text{ mm}, \ L_0 = 12.1 \text{ mm}, \ N_I = 38 \text{ coils}.\)

\[
D = OD - d = 0.95 - 0.25 = 0.7 \text{ mm}
\]

Eq. (10-1): \[ C = \frac{D}{d} = 0.7/0.25 = 2.8 \quad \text{(low)} \]

Eq. (10-5): \[ K_B = \frac{4C + 2}{4C - 3} = \frac{4(2.8) + 2}{4(2.8) - 3} = 1.610 \]
Table 10-1: \( N_a = N_t - 2 = 38 - 2 = 36 \) coils (high)

Table 10-5: \( G = 69.0 \times 10^3 \) MPa.

Eq. (10-9): 
\[
k = \frac{d^4G}{8D^3N_a} = \frac{0.25^4 (69.0) \times 10^3}{8(0.7)^3 36} = 2.728 \text{ N/mm}
\]

Table 10-1: 
\( L_x = dN_t = 0.25(38) = 9.5 \) mm
\( y_s = L_0 - L_x = 12.1 - 9.5 = 2.6 \) mm
\( F_s = k y_s = 2.728(2.6) = 7.093 \) N

Eq. (10-7): 
\[
\tau_s = K_B \frac{8FD}{\pi d^3} = 1.610 \frac{8(7.093)0.7}{(0.25^3)} = 1303 \text{ MPa}
\]

Table 10-4 (dia. less than table): \( A = 1867 \text{ MPa-mm}^m \), \( m = 0.146 \)

Eq. (10-14): 
\[
S_{st} = \frac{A}{d_m} = \frac{1867}{0.25^{0.146}} = 2286 \text{ MPa}
\]

Table 10-6: \( S_{sy} = 0.35 S_{st} = 0.35(2286) = 734 \text{ MPa} \)

\( \tau_s > S_{sy} \), that is, 1303 > 734 MPa, the spring is not solid-safe. Return to Eq. (1) with 
\( F_s = k y_s \) and \( \tau_s = S_{sy}/n_s \), and solve for \( y_s \), giving 
\[
y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B kD} = \frac{(734/1.2)\pi (0.25^3)}{8(1.610)2.728(0.7)} = 1.22 \text{ mm}
\]

The free length should be wound to
\[
L_0 = L_x + y_s = 9.5 + 1.22 = 10.72 \text{ mm} \quad \text{Ans.}
\]

This only addresses the solid-safe criteria. There are additional problems.

10-16 
Given: A228 music wire, squared and ground ends, \( d = 1.2 \) mm, OD = 6.5 mm, \( L_0 = 15.7 \) mm, \( N_t = 10.2 \) coils.

\( D = \text{OD} - d = 6.5 - 1.2 = 5.3 \) mm

Eq. (10-1): 
\( C = D/d = 5.3/1.2 = 4.417 \)

Eq. (10-5): 
\[
K_B = \frac{4C + 2}{4C - 3} = \frac{4(4.417) + 2}{4(4.417) - 3} = 1.368
\]

Table (10-1): \( N_a = N_t - 2 = 10.2 - 2 = 8.2 \) coils

Table 10-5 (\( d = 1.2/25.4 = 0.0472 \) in): 
\( G = 81.7 \times 10^3 \) MPa.

Eq. (10-9): 
\[
k = \frac{d^4G}{8D^3N_a} = \frac{1.2^4(81.7) \times 10^3}{8(5.3^3)8.2} = 17.35 \text{ N/mm}
\]

Table 10-1: 
\( L_x = dN_t = 1.2(10.2) = 12.24 \) mm
\( y_s = L_0 - L_x = 15.7 - 12.24 = 3.46 \) mm
\( F_s = k y_s = 17.35(3.46) = 60.03 \) N
Given: A229 OQ&T steel, squared and ground ends, \(d = 3.5\) mm, OD = 50.6 mm, \(L_0 = 75.5\) mm, \(N_t = 5.5\) coils.

\[
D = \text{OD} - d = 50.6 - 3.5 = 47.1\ \text{mm}
\]

Eq. (10-1): \(C = D/d = 47.1/3.5 = 13.46\) (high)

Eq. (10-5): \(K_B = \frac{4C + 2}{4C - 3} = \frac{4(13.46) + 2}{4(13.46) - 3} = 1.098\)

Table 10-1: \(N_a = N_t - 2 = 5.5 - 2 = 3.5\) coils

A229 OQ&T steel is not given in Table 10-5. From Table A-5, for carbon steels, \(G = 79.3(10^3)\) MPa.

Eq. (10-9): \(k = \frac{d^4G}{8D^3N_a} = \frac{3.5^4(79.3)10^3}{8(47.1^3)3.5} = 4.067\ \text{N/mm}\)

Table 10-1: \(L_s = dN_t = 3.5(5.5) = 19.25\ \text{mm}\)
\(y_s = L_0 - L_s = 75.5 - 19.25 = 56.25\ \text{mm}\)
\(F_s = ky_s = 4.067(56.25) = 228.8\ \text{N}\)

Eq. (10-7): \(\tau_s = K_B \frac{8F_D}{\pi d^3} = 1.098 \frac{8(228.8)47.1}{\pi (3.5^3)} = 702.8\ \text{MPa} (1)\)

Table 10-4: \(A = 1855\ \text{MPa-mm}^m, \ m = 0.187\)

Eq. (10-14): \(S_{ut} = \frac{A}{d^m} = \frac{1855}{3.5^{0.187}} = 1468\ \text{MPa}\)

Table 10-6: \(S_{sy} = 0.50 S_{ut} = 0.50(1468) = 734\ \text{MPa}\)
\(n_s = \frac{S_{sy}}{\tau_s} = \frac{734}{702.8} = 1.04\) Spring is not solid-safe \((n_s < 1.2)\)

Return to Eq. (1) with \(F_s = ky_s\) and \(\tau_s = S_{sy}/n_s\), and solve for \(y_s\), giving
\[
y_s = \frac{\left(S_{sy}/n_s\right)\pi d^3}{8K_kkd} = \frac{\left(734/1.2\right)\pi (3.5^3)}{8(1.098)4.067(47.1)} = 48.96\ \text{mm}
\]

The free length should be wound to
\[ L_0 = L_s + y_s = 19.25 + 48.96 = 68.2 \text{ mm} \quad \text{Ans.} \]

10-18  Given: B159 phosphor-bronze, squared and ground ends, \( d = 3.8 \text{ mm}, \ OD = 31.4 \text{ mm}, \ L_0 = 71.4 \text{ mm}, \ N_t = 12.8 \text{ coils.} \)

\[ D = OD - d = 31.4 - 3.8 = 27.6 \text{ mm} \]

Eq. (10-1): \[ C = D/d = 27.6/3.8 = 7.263 \]

Eq. (10-5): \[ K_B = \frac{4C + 2}{4C - 3} = \frac{4(7.263) + 2}{4(7.263) - 3} = 1.192 \]

Table 10-1: \[ N_a = N_t - 2 = 12.8 - 2 = 10.8 \text{ coils} \]

Table 10-5: \[ G = 41.4(10^3) \text{ MPa.} \]

Eq. (10-9): \[ k = \frac{d^4G}{8D^3N_a} = \frac{3.8^4(41.4)10^3}{8(27.6^3)10.8} = 4.752 \text{ N/mm} \]

Table 10-1: \[ L_s = dN_t = 3.8(12.8) = 48.64 \text{ mm} \]

\[ y_s = L_0 - L_s = 71.4 - 48.64 = 22.76 \text{ mm} \]

\[ F_s = ky_s = 4.752(22.76) = 108.2 \text{ N} \]

Eq. (10-7): \[ \tau_s = K_B \frac{8F_sD}{\pi d^3} = 1.192 \frac{8(108.2)27.6}{\pi(3.8^3)} = 165.2 \text{ MPa} \quad (1) \]

Table 10-4 \( (d = 3.8/25.4 = 0.150 \text{ in}): \ A = 932 \text{ MPa-mm}^m, \ m = 0.064 \)

Eq. (10-14): \[ S_{ut} = \frac{A}{d^m} = \frac{932}{3.8^{0.064}} = 855.7 \text{ MPa} \]

Table 10-6: \[ S_{sy} = 0.35S_{ut} = 0.35(855.7) = 299.5 \text{ MPa} \]

\[ n_s = \frac{S_{sy}}{\tau_s} = \frac{299.5}{165.2} = 1.81 \quad \text{Spring is solid-safe} \ (n_s > 1.2) \quad \text{Ans.} \]

10-19  Given: A232 chrome-vanadium steel, squared and ground ends, \( d = 4.5 \text{ mm}, \ OD = 69.2 \text{ mm}, \ L_0 = 215.6 \text{ mm}, \ N_t = 8.2 \text{ coils.} \)

\[ D = OD - d = 69.2 - 4.5 = 64.7 \text{ mm} \]

Eq. (10-1): \[ C = D/d = 64.7/4.5 = 14.38 \text{ (high)} \]

Eq. (10-5): \[ K_B = \frac{4C + 2}{4C - 3} = \frac{4(14.38) + 2}{4(14.38) - 3} = 1.092 \]

Table 10-1: \[ N_a = N_t - 2 = 8.2 - 2 = 6.2 \text{ coils} \]

Table 10-5: \[ G = 77.2(10^3) \text{ MPa.} \]

Eq. (10-9): \[ k = \frac{d^4G}{8D^3N_a} = \frac{4.5^4(77.2)10^3}{8(64.7^3)6.2} = 2.357 \text{ N/mm} \]

Table 10-1: \[ L_s = dN_t = 4.5(8.2) = 36.9 \text{ mm} \]
\[ y_s = L_0 - L_s = 215.6 - 36.9 = 178.7 \text{ mm} \]

\[ F_s = k y_s = 2.357(178.7) = 421.2 \text{ N} \]

Eq. (10-7): \[ \tau_s = K_n \frac{8F_D}{\pi d^3} = 1.092 \frac{8(421.2)(64.7)}{\pi (4.5^3)} = 832 \text{ MPa} \] (1)

\[ \text{Eq. (10-14):} \quad S_{ut} = \frac{A}{d^m} = \frac{2005}{4.5^{0.168}} = 1557 \text{ MPa} \]

Table 10-6: \[ S_{sy} = 0.50 \quad S_{ut} = 0.50(1557) = 779 \text{ MPa} \]

\[ \tau_s > S_{sy}, \text{ that is, } 832 > 779 \text{ MPa}, \text{ the spring is not solid-safe. Return to Eq. (1) with} \]

\[ F_s = k y_s \text{ and } \tau_s = S_{sy}/n_s, \text{ and solve for } y_s, \text{ giving} \]

\[ y_s = \frac{\left( S_{sy}/n_s \right) \pi d^3}{8K_n k D} = \frac{\left( 779/1.2 \right) \pi (4.5^3)}{8(1.092)(2.357)(64.7)} = 139.5 \text{ mm} \]

The free length should be wound to

\[ L_0 = L_s + y_s = 36.9 + 139.5 = 176.4 \text{ mm} \quad \text{Ans.} \]

This only addresses the solid-safe criteria. There are additional problems.

---

**10-20**

Given: A227 HD steel.

From the figure: \( L_0 = 4.75 \text{ in}, \ OD = 2 \text{ in}, \text{ and } d = 0.135 \text{ in}. \) Thus

\[ D = OD - d = 2 - 0.135 = 1.865 \text{ in} \]

(a) By counting, \( N_t = 12.5 \text{ coils}. \) Since the ends are squared along 1/4 turn on each end,

\[ N_a = 12.5 - 0.5 = 12 \text{ turns} \quad \text{Ans.} \]

\[ p = 4.75 / 12 = 0.396 \text{ in} \quad \text{Ans.} \]

The solid stack is 13 wire diameters

\[ L_s = 13(0.135) = 1.755 \text{ in} \quad \text{Ans.} \]

(b) From Table 10-5, \( G = 11.4 \text{ Mpsi} \)

\[ k = \frac{d^4G}{8D^3 N_a} = \frac{0.135^4(11.4)(10^6)}{8(1.865^3)(12)} = 6.08 \text{ lbf/in} \quad \text{Ans.} \]

(c) \( F_s = k(L_0 - L_a) = 6.08(4.75 - 1.755) = 18.2 \text{ lbf} \quad \text{Ans.} \)

(d) \( C = D/d = 1.865/0.135 = 13.81 \)
\[ K_B = \frac{4(13.81) + 2}{4(13.81) - 3} = 1.096 \]
\[ \tau_s = K_B \times \frac{8FD}{\pi d^3} = 1.096 \times \frac{8(18.2)(1.865)}{\pi (0.135^3)} = 38.5\left(10^3\right) \text{ psi} = 38.5 \text{ kpsi} \quad \text{Ans.} \]

10-21 For the wire diameter analyzed, \( G = 11.75 \text{ Mpsi} \) per Table 10-5. Use squared and ground ends. The following is a spreadsheet study using Fig. 10-3 for parts (a) and (b). For \( N_a \), \( k = F_{max}/y = 20/2 = 10 \text{ lbf/in.} \) For \( \tau_s \), \( F_s = 20(1 + \xi) = 20(1 + 0.15) = 23 \text{ lbf.} \)

<table>
<thead>
<tr>
<th>Source</th>
<th>Parameter</th>
<th>Values</th>
<th>Source</th>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eq. (10-1)</td>
<td>( C )</td>
<td>11.667</td>
<td>Eq. (10-1)</td>
<td>( C )</td>
<td>11.667</td>
</tr>
<tr>
<td>Eq. (10-9)</td>
<td>( N_a )</td>
<td>6.937</td>
<td>Eq. (10-9)</td>
<td>( N_a )</td>
<td>6.937</td>
</tr>
<tr>
<td>Table 10-1</td>
<td>( N_c )</td>
<td>8.937</td>
<td>Table 10-1</td>
<td>( N_c )</td>
<td>8.937</td>
</tr>
<tr>
<td>Table 10-1</td>
<td>( L_s )</td>
<td>0.670</td>
<td>Table 10-1</td>
<td>( L_s )</td>
<td>0.670</td>
</tr>
<tr>
<td>1.15y + ( L_0 )</td>
<td>( L_0 )</td>
<td>2.970</td>
<td>1.15y + ( L_0 )</td>
<td>( L_0 )</td>
<td>2.970</td>
</tr>
<tr>
<td>Eq. (10-13)</td>
<td>( L_0/c_0 )</td>
<td>4.603</td>
<td>Eq. (10-13)</td>
<td>( L_0/c_0 )</td>
<td>4.603</td>
</tr>
<tr>
<td>Table 10-4</td>
<td>( A )</td>
<td>201.000</td>
<td>Table 10-4</td>
<td>( A )</td>
<td>201.000</td>
</tr>
<tr>
<td>Table 10-4</td>
<td>( m )</td>
<td>0.145</td>
<td>Table 10-4</td>
<td>( m )</td>
<td>0.145</td>
</tr>
<tr>
<td>Eq. (10-14)</td>
<td>( S_w )</td>
<td>292.626</td>
<td>Eq. (10-14)</td>
<td>( S_w )</td>
<td>292.626</td>
</tr>
<tr>
<td>Table 10-6</td>
<td>( S_{sy} )</td>
<td>131.681</td>
<td>Table 10-6</td>
<td>( S_{sy} )</td>
<td>131.681</td>
</tr>
<tr>
<td>Eq. (10-5)</td>
<td>( K_B )</td>
<td>1.115</td>
<td>Eq. (10-5)</td>
<td>( K_B )</td>
<td>1.115</td>
</tr>
<tr>
<td>Eq. (10-7)</td>
<td>( \xi )</td>
<td>135.335</td>
<td>Eq. (10-7)</td>
<td>( \xi )</td>
<td>135.335</td>
</tr>
<tr>
<td>Eq. (10-3)</td>
<td>( n_s )</td>
<td>0.973</td>
<td>Eq. (10-3)</td>
<td>( n_s )</td>
<td>0.973</td>
</tr>
<tr>
<td>Eq. (10-22)</td>
<td>fom</td>
<td>-0.282</td>
<td>Eq. (10-22)</td>
<td>fom</td>
<td>-0.282</td>
</tr>
</tbody>
</table>

For \( n_s \geq 1.2 \), the optimal size is \( d = 0.085 \) in for both cases.

10-22 In Prob. 10-21, there is an advantage of first selecting \( d \) as one can select from the available sizes (Table A-28). Selecting \( C \) first requires a calculation of \( d \) where then a size must be selected from Table A-28.

Consider part (a) of the problem. It is required that

\[ \text{ID} = D - d = 0.800 \text{ in.} \quad (1) \]

From Eq. (10-1), \( D = Cd \). Substituting this into the first equation yields

\[ d = \frac{0.800}{C - 1} \quad (2) \]

Starting with \( C = 10 \), from Eq. (2) we find that \( d = 0.089 \) in. From Table A-28, the closest diameter is \( d = 0.090 \) in. Substituting this back into Eq. (1) gives \( D = 0.890 \) in, with \( C = 0.890/0.090 = 9.889 \), which are acceptable. From this point the solution is the same as Prob. 10-21. For part (b), use
\[
\text{OD} = D + d = 0.950 \text{ in.} \quad (3)
\]

and,
\[
d = \frac{0.800}{C - 1} \quad (4)
\]

<table>
<thead>
<tr>
<th>Source</th>
<th>Parameter</th>
<th>Values</th>
<th>Source</th>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eq. (2)</td>
<td>(d)</td>
<td>0.089</td>
<td>Eq. (4)</td>
<td>(d)</td>
<td>0.086</td>
</tr>
<tr>
<td>Table A-28</td>
<td>(d)</td>
<td>0.090</td>
<td></td>
<td>Table A-28</td>
<td>(d)</td>
</tr>
<tr>
<td>Eq. (1)</td>
<td>(D)</td>
<td>0.890</td>
<td>Eq. (3)</td>
<td>(D)</td>
<td>0.865</td>
</tr>
<tr>
<td>Eq. (10-1)</td>
<td>(C)</td>
<td>9.889</td>
<td>Eq. (10-1)</td>
<td>(C)</td>
<td>10.176</td>
</tr>
<tr>
<td>Eq. (10-9)</td>
<td>(N_e)</td>
<td>13.669</td>
<td>Eq. (10-9)</td>
<td>(N_e)</td>
<td>11.846</td>
</tr>
<tr>
<td>Table 10-1</td>
<td>(N_e)</td>
<td>15.669</td>
<td>Table 10-1</td>
<td>(N_e)</td>
<td>13.846</td>
</tr>
<tr>
<td></td>
<td>(L)</td>
<td>1.410</td>
<td></td>
<td>(L)</td>
<td>1.177</td>
</tr>
<tr>
<td>1.15y + (L)</td>
<td>(L_0)</td>
<td>3.710</td>
<td>1.15y + (L)</td>
<td>(L_0)</td>
<td>3.477</td>
</tr>
<tr>
<td>Eq. (10-13)</td>
<td>((L_0)_{ax})</td>
<td>4.681</td>
<td>Eq. (10-13)</td>
<td>((L_0)_{ax})</td>
<td>4.550</td>
</tr>
<tr>
<td>Table 10-4</td>
<td>(A)</td>
<td>201.000</td>
<td>Table 10-4</td>
<td>(A)</td>
<td>201.000</td>
</tr>
<tr>
<td>Table 10-4</td>
<td>(m)</td>
<td>0.145</td>
<td>Table 10-4</td>
<td>(m)</td>
<td>0.145</td>
</tr>
<tr>
<td>Eq. (10-14)</td>
<td>(S_{ax})</td>
<td>284.991</td>
<td>Eq. (10-14)</td>
<td>(S_{ax})</td>
<td>287.363</td>
</tr>
<tr>
<td>Table 10-6</td>
<td>(S_{ax})</td>
<td>128.246</td>
<td>Table 10-6</td>
<td>(S_{ax})</td>
<td>129.313</td>
</tr>
<tr>
<td>Eq. (10-5)</td>
<td>(K_B)</td>
<td>1.135</td>
<td>Eq. (10-5)</td>
<td>(K_B)</td>
<td>1.135</td>
</tr>
<tr>
<td>Eq. (10-7)</td>
<td>(\tau_s)</td>
<td>81.167</td>
<td>Eq. (10-7)</td>
<td>(\tau_s)</td>
<td>93.643</td>
</tr>
<tr>
<td>(n_s = S_{ax}/\tau_s)</td>
<td>(n_s)</td>
<td>1.580</td>
<td>(n_s = S_{ax}/\tau_s)</td>
<td>(n_s)</td>
<td>1.381</td>
</tr>
<tr>
<td>Eq. (10-22)</td>
<td>fom</td>
<td>-0.725</td>
<td>Eq. (10-22)</td>
<td>fom</td>
<td>-0.555</td>
</tr>
</tbody>
</table>

Again, for \(n_s \geq 1.2\), the optimal size is \(= 0.085\) in.

Although this approach used less iterations than in Prob. 10-21, this was due to the initial values picked and not the approach.

10-23 One approach is to select A227 HD steel for its low cost. Try \(L_0 = 48\) mm, then for \(y = 48 - 37.5 = 10.5\) mm when \(F = 45\) N. The spring rate is \(k = F/y = 45/10.5 = 4.286\) N/mm.

For a clearance of 1.25 mm with screw, \(\text{ID} = 10 + 1.25 = 11.25\) mm. Starting with \(d = 2\) mm,
\[
D = \text{ID} + d = 11.25 + 2 = 13.25 \text{ mm}
\]

\[
C = D/d = 13.25/2 = 6.625 \quad \text{(acceptable)}
\]

Table 10-5 \((d = 2/25.4 = 0.0787\) in\.): \(G = 79.3\) GPa

\[
\text{Eq. (10-9): } N_a = \frac{d^4G}{8kD^3} = \frac{2^4(79.3)10^3}{8(4.286)13.25^3} = 15.9 \text{ coils}
\]

Assume squared and closed.
Table 10-1: \[ N_t = N_a + 2 = 15.9 + 2 = 17.9 \text{ coils} \]
\[ L_s = dN_t = 2(17.9) = 35.8 \text{ mm} \]

\[ y_s = L_0 - L_s = 48 - 35.8 = 12.2 \text{ mm} \]
\[ F_s = ky_s = 4.286(12.2) = 52.29 \text{ N} \]

Eq. (10-5):
\[ K_B = \frac{4C + 2}{4C - 3} = \frac{4(6.625) + 2}{4(6.625) - 3} = 1.213 \]

Eq. (10-7):
\[ \tau_s = K_B \frac{8FD}{\pi d^3} = 1.213 \left[ \frac{8(52.29)(13.25)}{\pi \left( \frac{2}{3} \right)^3} \right] = 267.5 \text{ MPa} \]

Table 10-4: \[ A = 1783 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.190 \]

Eq. (10-14):
\[ S_{ut} = \frac{A}{d^m} = \frac{1783}{2^{0.190}} = 1563 \text{ MPa} \]

Table 10-6:
\[ S_{sy} = 0.45S_{ut} = 0.45(1563) = 703.3 \text{ MPa} \]
\[ n_s = \frac{S_{sy}}{\tau_s} = \frac{703.3}{267.5} = 2.63 > 1.2 \quad \text{O.K.} \]

No other diameters in the given range work. So specify

A227-47 HD steel, \( d = 2 \text{ mm}, \quad D = 13.25 \text{ mm}, \quad \text{ID} = 11.25 \text{ mm}, \quad \text{OD} = 15.25 \text{ mm}, \) squared and closed, \( N_t = 17.9 \text{ coils}, \quad N_a = 15.9 \text{ coils}, \quad k = 4.286 \text{ N/mm}, \quad L_s = 35.8 \text{ mm}, \) and \( L_0 = 48 \text{ mm}. \quad \text{Ans.} \)

---

10-24 Select A227 HD steel for its low cost. Try \( L_0 = 48 \text{ mm}, \) then for \( y = 48 - 37.5 = 10.5 \text{ mm} \) when \( F = 45 \text{ N}. \) The spring rate is \( k = F/y = 45/10.5 = 4.286 \text{ N/mm}. \)

For a clearance of 1.25 mm with screw, \( \text{ID} = 10 + 1.25 = 11.25 \text{ mm}. \)

\[ D - d = 11.25 \quad (1) \]
and,
\[ D = Cd \quad (2) \]

Starting with \( C = 8, \) gives \( D = 8d. \) Substitute into Eq. (1) resulting in \( d = 1.607 \text{ mm}. \)
Selecting the nearest diameter in the given range, \( d = 1.6 \text{ mm}. \) From this point, the calculations are shown in the third column of the spreadsheet output shown. We see that for \( d = 1.6 \text{ mm}, \) the spring is not solid safe. Iterating on \( C \) we find that \( C = 6.5 \) provides acceptable results with the specifications

A227-47 HD steel, \( d = 2 \text{ mm}, \quad D = 13.25 \text{ mm}, \quad \text{ID} = 11.25 \text{ mm}, \quad \text{OD} = 15.25 \text{ mm}, \) squared and closed, \( N_t = 17.9 \text{ coils}, \quad N_a = 15.9 \text{ coils}, \quad k = 4.286 \text{ N/mm}, \quad L_s = 35.8 \text{ mm}, \) and \( L_0 = 48 \text{ mm}. \quad \text{Ans.} \)
The only difference between selecting $C$ first rather than $d$ as was done in Prob. 10-23, is that once $d$ is calculated, the closest wire size must be selected. Iterating on $d$ uses available wire sizes from the beginning.

### 10-25
A stock spring catalog may have over two hundred pages of compression springs with up to 80 springs per page listed.
- Students should be made aware that such catalogs exist.
- Many springs are selected from catalogs rather than designed.
- The wire size you want may not be listed.
- Catalogs may also be available on disk or the web through search routines. For example, disks are available from Century Spring at 1 - (800) - 237 - 5225 www.centuryspring.com
- It is better to familiarize yourself with vendor resources rather than invent them yourself.
- Sample catalog pages can be given to students for study.

### 10-26
Given: ID = 0.6 in, $C = 10$, $L_0 = 5$ in, $L_s = 5 - 3 = 2$ in, sq. & grd ends, unpeened, HD A227 wire.

(a) With ID = $D - d = 0.6$ in and $C = D/d = 10 \Rightarrow 10 d - d = 0.6 \Rightarrow d = 0.0667$ in \ Ans., and $D = 0.667$ in.

(b) Table 10-1: \[ L_s = dN_t = 2 \text{ in} \Rightarrow N_t = 2/0.0667 = 30 \text{ coils} \ \text{Ans.} \]

(c) Table 10-1: \[ N_a = N_t - 2 = 30 - 2 = 28 \text{ coils} \]
Table 10-5: \( G = 11.5 \) Mpsi

Eq. (10-9): \( k = \frac{d^4 G}{8D^3 N_u} = \frac{0.0667^4 (11.5)10^6}{8(0.667)^3} \) 28 = 3.424 lbf/in \( \text{Ans.} \)

(d) Table 10-4: \( A = 140 \) kpsi-in\(^m\), \( m = 0.190 \)

Eq. (10-14): \( S_{ut} = \frac{A}{d_m} = \frac{140}{0.0667^{0.190}} = 234.2 \) kpsi

Table 10-6: \( S_{sy} = 0.45 \) \( S_{ut} = 0.45 \) (234.2) = 105.4 kpsi

Eq. (10-5): \( F_s = ky_s = 3.424(3) = 10.27 \) lbf

Eq. (10-7):
\[
\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.135 \frac{8(10.27)0.667}{\pi(0.0667^3)} = 66.72 \left(10^3\right) \text{ psi} = 66.72 \text{ kpsi}
\]
\[
n_s = \frac{S_{sy}}{\tau_s} = \frac{105.4}{66.72} = 1.58 \text{ Ans.}
\]

(e) \( \tau_s = \tau_m = 0.5 \tau_s = 0.5(66.72) = 33.36 \) kpsi, \( r = \tau_u / \tau_m = 1 \). Using the Gerber fatigue failure criterion with Zimmerli data,

Eq. (10-30):
\[
S_{su} = 0.67 S_{ut} = 0.67(234.2) = 156.9 \text{ kpsi}
\]

The Gerber ordinate intercept for the Zimmerli data is
\[
S_{se} = \frac{S_{su}}{1 - \left( \frac{S_{su}}{S_{sa}} \right)^2} = \frac{35}{1 - \left( \frac{55}{156.9} \right)^2} = 39.9 \text{ kpsi}
\]

Table 6-7, p. 315,
\[
S_{sa} = \frac{r^2 S_{su}^2}{2S_{se}} \left[ -1 + \sqrt{1 + \left( \frac{2S_{se}}{rS_{su}} \right)^2} \right]
\]
\[
= \frac{1^2 (156.9)^2}{2(39.9)} \left[ -1 + \sqrt{1 + \left( \frac{2(39.9)}{1(156.9)} \right)^2} \right] = 37.61 \text{ kpsi}
\]
\[
n_f = \frac{S_{su}}{\tau_s} = \frac{37.61}{33.36} = 1.13 \text{ Ans.}
\]

\[\text{10-27 Given: OD } \leq 0.9 \text{ in, } C = 8, L_0 = 3 \text{ in, } L_s = 1 \text{ in, } y_s = 3 - 1 = 2 \text{ in, sq. ends, unpeened, music wire.} \]

(a) Try OD = \( D + d = 0.9 \) in, \( C = D/d = 8 \Rightarrow D = 8d \Rightarrow 9d = 0.9 \Rightarrow d = 0.1 \text{ Ans.} \]
\[ D = 8(0.1) = 0.8 \text{ in} \]

(b) Table 10-1: \[ L_s = d (N_r + 1) \Rightarrow N_r = L_s / d - 1 = 1/0.1 - 1 = 9 \text{ coils} \quad \text{Ans.} \]

Table 10-1: \[ N_a = N_r - 2 = 9 - 2 = 7 \text{ coils} \]

(c) Table 10-5: \[ G = 11.75 \text{ Mpsi} \]

Eq. (10-9): \[ k = \frac{d^4 G}{8D^3 N_a} = \frac{0.1^4 (11.75)10^6}{8(0.8)^3} = 40.98 \text{ lbf/in} \quad \text{Ans.} \]

(d) \[ F_s = ky_s = 40.98(2) = 81.96 \text{ lbf} \]

Eq. (10-5): \[ K_B = \frac{4C + 2}{4C - 3} = \frac{4(8) + 2}{4(8) - 3} = 1.172 \]

Eq. (10-7): \[ \tau_s = K_B \frac{8F_D}{\pi d^3} = 1.172 \frac{8(81.96)0.8}{\pi (0.1)} = 195.7 \left(10^3\right) \text{ psi} = 195.7 \text{ kpsi} \]

Table 10-4: \[ A = 201 \text{ kpsi-in}^m, \quad m = 0.145 \]

Eq. (10-14): \[ S_{ut} = \frac{A}{d^m} = \frac{201}{0.1^{0.145}} = 280.7 \text{ kpsi} \]

Table 10-6: \[ S_{sy} = 0.45 \quad S_{ut} = 0.45(280.7) = 126.3 \text{ kpsi} \]

\[ n_s = \frac{S_{sy}}{\tau_s} = \frac{126.3}{195.7} = 0.645 \quad \text{Ans.} \]

(e) \[ \tau_s = \tau_m = \frac{\tau_s}{2} = 97.85 \text{ kpsi} \]. Using the Gerber fatigue failure criterion with Zimmerli data,

Eq. (10-30): \[ S_{su} = 0.67 S_{ut} = 0.67(280.7) = 188.1 \text{ kpsi} \]

The Gerber ordinate intercept for the Zimmerli data is

\[ S_{se} = \frac{S_{su}}{1 - (S_{so} / S_{su})^2} = \frac{35}{1 - (55/188.1)^2} = 38.3 \text{ kpsi} \]

Table 6-7, p. 315,

\[ S_{so} = \frac{r^2 S_{su}^2}{2S_{se}} \left[ -1 + \sqrt{1 + \left( \frac{2S_{se}}{rS_{su}} \right)^2} \right] \]

\[ = \frac{1^2 (188.1)^2}{2(38.3)} \left[ -1 + \sqrt{1 + \left( \frac{2(38.3)}{1(188.1)} \right)^2} \right] = 36.83 \text{ kpsi} \]
\[ n_f \frac{S_{ut}}{\tau_s} = \frac{36.83}{97.85} = 0.376 \quad \text{Ans.} \]

Obviously, the spring is severely under designed and will fail statically and in fatigue. Increasing \( C \) would improve matters. Try \( C = 12 \). This yields \( n_s = 1.83 \) and \( n_f = 1.00 \).

**10-28** Given: \( F_{\text{max}} = 300 \text{ lbf}, \) \( F_{\text{min}} = 150 \text{ lbf}, \) \( \Delta y = 1 \text{ in}, \) \( \text{OD} = 2.1 - 0.2 = 1.9 \text{ in}, \) \( C = 7, \) unpeened, squared & ground, oil-tempered wire.

(a) \[ D = \text{OD} - d = 1.9 - d \quad (1) \]
\[ C = \frac{D}{d} = 7 \quad \Rightarrow \quad D = 7d \quad (2) \]
Substitute Eq. (2) into (1)
\[ 7d = 1.9 - d \quad \Rightarrow \quad d = 1.9/8 = 0.2375 \text{ in} \quad \text{Ans.} \]

(b) From Eq. (2):
\[ D = 7d = 7(0.2375) = 1.663 \text{ in} \quad \text{Ans.} \]

(c) \[ k = \frac{\Delta F}{\Delta y} = \frac{300 - 150}{1} = 150 \text{ lbf/in} \quad \text{Ans.} \]

(d) Table 10-5:
\[ G = 11.6 \text{ Mpsi} \]
Eq. (10-9):
\[ N_a = \frac{d^4G}{8D^3k} = \frac{0.2375^4(11.6)10^6}{8(1.663^3)150} = 6.69 \text{ coils} \]
Table 10-1:
\[ N_t = N_a + 2 = 8.69 \text{ coils} \quad \text{Ans.} \]

(e) Table 10-4:
\[ A = 147 \text{ kpsi-in}^m, \quad m = 0.187 \]
Eq. (10-14):
\[ S_{ut} = \frac{A}{d^m} = \frac{147}{0.2375^{0.187}} = 192.3 \text{ kpsi} \]
Table 10-6:
\[ S_{sy} = 0.5 S_{ut} = 0.5(192.3) = 96.15 \text{ kpsi} \]
Eq. (10-5):
\[ K_B = \frac{4C + 2}{4C - 3} = \frac{4(7) + 2}{4(7) - 3} = 1.2 \]
Eq. (10-7):
\[ \tau_s = K_B \frac{8F_sD}{\pi d^3} = S_{sy} \]
\[ F_t = \frac{\pi d^3 s_n}{8 K_y D} = \frac{\pi \left(0.2375^3\right) 96.15 \left(10^3\right)}{8(1.2)1.663} = 253.5 \text{ lbf} \]

\[ y_s = F_t / k = 253.5/150 = 1.69 \text{ in} \]

Table 10-1:
\[ L_s = N_t d = 8.46(0.2375) = 2.01 \text{ in} \]
\[ L_0 = L_s + y_s = 2.01 + 1.69 = 3.70 \text{ in} \quad \text{Ans.} \]

### 10-29
For a coil radius given by:
\[ R = R_i + \frac{R_2 - R_1}{2\pi N} \theta \]

The torsion of a section is \( T = PR \) where \( dL = R \, d\theta \)

\[ \delta_p = \frac{\partial U}{\partial P} = \frac{1}{GJ} \int_{0}^{2\pi N} \frac{T}{\partial P} \, dL = \frac{1}{GJ} \int_{0}^{2\pi N} PR^3 \, d\theta \]
\[ = \frac{P}{GJ} \int_{0}^{2\pi N} \left( R_i + \frac{R_2 - R_1}{2\pi N} \theta \right)^3 \, d\theta \]
\[ = \frac{P}{GJ} \left( \frac{1}{4} \pi PN \left( R_2 - R_i \right) \left[ \left( R_1 + \frac{R_2 - R_1}{2\pi N} \theta \right)^4 \right]_0^{2\pi N} \right) \]
\[ = \frac{\pi PN}{2GJ(R_2 - R_i)} \left( R_1^4 - R_i^4 \right) = \frac{\pi PN}{2GJ} (R_1 + R_2) \left( R_1^2 + R_2^2 \right) \]
\[ J = \frac{\pi}{32} d^4 \quad \therefore \delta_p = \frac{16PN}{Gd^4} (R_1 + R_2) \left( R_1^2 + R_2^2 \right) \]

\[ k = \frac{P}{\delta_p} = \frac{d^4 G}{16N(R_1 + R_2) \left( R_1^2 + R_2^2 \right)} \quad \text{Ans.} \]

### 10-30
Given: \( F_{\text{min}} = 4 \text{ lbf}, F_{\text{max}} = 18 \text{ lbf}, k = 9.5 \text{ lbf/in}, \) OD \( \leq 2.5 \text{ in}, n_f = 1.5. \)

For a food service machinery application select A313 Stainless wire.

Table 10-5: \( G = 10(10^9) \text{ psi} \)

Note that for 
\[ 0.013 \leq d \leq 0.10 \text{ in} \quad A = 169, \quad m = 0.146 \]
\[ 0.10 < d \leq 0.20 \text{ in} \quad A = 128, \quad m = 0.263 \]
\[ F_u = \frac{18 - 4}{2} = 7 \text{ lbf}, \quad F_m = \frac{18 + 4}{2} = 11 \text{ lbf}, \quad r = 7 / 11 \]

Try, \( d = 0.080 \text{ in}, \quad S_{ut} = \frac{169}{(0.08)^{0.146}} = 244.4 \text{ kpsi} \)
\( S_{sa} = 0.67 S_{ut} = 163.7 \text{ kpsi}, \quad S_{sy} = 0.35 S_{ut} = 85.5 \text{ kpsi} \)

Try unpeened using Zimmerli’s endurance data: \( S_{sa} = 35 \text{ kpsi}, S_{sm} = 55 \text{ kpsi} \)

Gerber:

\[
S_{se} = \frac{S_{sa}}{1 - (S_{sa} / S_{ut})^2} = \frac{35}{1 - (55 / 163.7)^2} = 39.5 \text{ kpsi}
\]

\[
S_{sa} = \frac{(7 / 11)^2(163.7)^2}{2(39.5)} \left\{ 1 + \left[ 1 + \left[ \frac{2(39.5)}{(7 / 11)(163.7)} \right]^2 \right] \right\} = 35.0 \text{ kpsi}
\]

\[
\alpha = S_{sa} / n_f = 35.0 / 1.5 = 23.3 \text{ kpsi}
\]

\[
\beta = \frac{8F_d}{\pi d^2}(10^{-3}) = \left[ \frac{8(7)}{\pi(0.08^2)} \right](10^{-3}) = 2.785 \text{ kpsi}
\]

\[
C = \frac{2(23.3) - 2.785}{4(2.785)} + \sqrt{\frac{2(23.3) - 2.785}{4(2.785)}}^2 - \frac{3(23.3)}{4(2.785)} = 6.97
\]

\[
D = C d = 6.97(0.08) = 0.558 \text{ in}
\]

\[
K_B = \frac{4C + 2}{4C - 3} = \frac{4(6.97) + 2}{4(6.97) - 3} = 1.201
\]

\[
\tau_a = K_B \left( \frac{8F_d D}{\pi d^3} \right) = 1.201 \left[ \frac{8(7)(0.558)}{\pi(0.08^2)} \right](10^{-3}) = 23.3 \text{ kpsi}
\]

\[
n_f = 35 / 23.3 = 1.50 \text{ checks}
\]

\[
N_a = \frac{Gd^4}{8kD^3} = \frac{10(10^5)(0.08)^4}{8(9.5)(0.558)^3} = 31.02 \text{ coils}
\]

\[
N_i = 31.02 + 2 = 33 \text{ coils} \quad L_s = dN_i = 0.08(33) = 2.64 \text{ in}
\]

\[
y_{max} = F_{max} / k = 18 / 9.5 = 1.895 \text{ in}
\]

\[
y_s = (1 + \xi)y_{max} = (1 + 0.15)(1.895) = 2.179 \text{ in}
\]

\[
L_o = 2.64 + 2.179 = 4.819 \text{ in}
\]

\[
(L_o)_{cr} = 2.63 \frac{D}{\alpha} = \frac{2.63(0.558)}{0.5} = 2.935 \text{ in}
\]

\[
\tau_s = 1.15(18 / 7)\tau_a = 1.15(18 / 7)(23.3) = 68.9 \text{ kpsi}
\]

\[
n_s = S_{sy} / \tau_s = 85.5 / 68.9 = 1.24
\]

\[
f = \sqrt{\frac{kg}{\pi^2 d^2 N_a \gamma}} = \sqrt{\frac{9.5(386)}{\pi^2(0.08^2)(0.558)(31.02)(0.283)}} = 109 \text{ Hz}
\]

These steps are easily implemented on a spreadsheet, as shown below, for different diameters.
The shaded areas depict conditions outside the recommended design conditions. Thus, one spring is satisfactory. The specifications are: A313 stainless wire, unpeened, squared and ground, $d = 0.0915$ in, OD = 0.879 + 0.092 = 0.971 in, $L_0 = 3.606$ in, and $N_t = 15.59$ turns

<table>
<thead>
<tr>
<th></th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>0.080</td>
<td>0.0915</td>
<td>0.1055</td>
<td>0.1205</td>
</tr>
<tr>
<td>$m$</td>
<td>0.146</td>
<td>0.146</td>
<td>0.263</td>
<td>0.263</td>
</tr>
<tr>
<td>$A$</td>
<td>169.000</td>
<td>169.000</td>
<td>128</td>
<td>128</td>
</tr>
<tr>
<td>$S_{at}$</td>
<td>244.363</td>
<td>239.618</td>
<td>231.257</td>
<td>223.311</td>
</tr>
<tr>
<td>$S_{sa}$</td>
<td>163.723</td>
<td>160.544</td>
<td>154.942</td>
<td>149.618</td>
</tr>
<tr>
<td>$S_{sy}$</td>
<td>85.527</td>
<td>83.866</td>
<td>80.940</td>
<td>78.159</td>
</tr>
<tr>
<td>$S_{se}$</td>
<td>39.452</td>
<td>39.654</td>
<td>40.046</td>
<td>40.469</td>
</tr>
<tr>
<td>$S_{sa}$</td>
<td>35.000</td>
<td>35.000</td>
<td>35.000</td>
<td>35.000</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>23.333</td>
<td>23.333</td>
<td>23.333</td>
<td>23.333</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.785</td>
<td>2.129</td>
<td>1.602</td>
<td>1.228</td>
</tr>
<tr>
<td>$C$</td>
<td>6.977</td>
<td>9.603</td>
<td>13.244</td>
<td>17.702</td>
</tr>
<tr>
<td>$D$</td>
<td>0.558</td>
<td>0.879</td>
<td>1.397</td>
<td>2.133</td>
</tr>
<tr>
<td>$K_B$</td>
<td>1.201</td>
<td>1.141</td>
<td>1.100</td>
<td>1.074</td>
</tr>
<tr>
<td>$\tau_a$</td>
<td>23.333</td>
<td>23.333</td>
<td>23.333</td>
<td>23.333</td>
</tr>
<tr>
<td>$n_f$</td>
<td>1.500</td>
<td>1.500</td>
<td>1.500</td>
<td>1.500</td>
</tr>
<tr>
<td>$N_a$</td>
<td>30.993</td>
<td>13.594</td>
<td>5.975</td>
<td>2.858</td>
</tr>
<tr>
<td>$N_b$</td>
<td>32.993</td>
<td>15.594</td>
<td>7.975</td>
<td>4.858</td>
</tr>
<tr>
<td>$L_S$</td>
<td>2.639</td>
<td>1.427</td>
<td>0.841</td>
<td>0.585</td>
</tr>
<tr>
<td>$y_s$</td>
<td>2.179</td>
<td>2.179</td>
<td>2.179</td>
<td>2.179</td>
</tr>
<tr>
<td>$L_0$</td>
<td>4.818</td>
<td>3.606</td>
<td>3.020</td>
<td>2.764</td>
</tr>
<tr>
<td>$(L_0)_{cr}$</td>
<td>2.936</td>
<td>4.622</td>
<td>7.350</td>
<td>11.220</td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>69.000</td>
<td>69.000</td>
<td>69.000</td>
<td>69.000</td>
</tr>
<tr>
<td>$n_s$</td>
<td>1.240</td>
<td>1.215</td>
<td>1.173</td>
<td>1.133</td>
</tr>
<tr>
<td>$f,(Hz)$</td>
<td>108.895</td>
<td>114.578</td>
<td>118.863</td>
<td>121.775</td>
</tr>
</tbody>
</table>

The steps are the same as in Prob. 10-30 except that the Gerber-Zimmerli criterion is replaced with Goodman-Zimmerli:

$$S_{sa} = \frac{S_{sa}}{1 - (S_{sm}/S_{sa})}$$

The problem then proceeds as in Prob. 10-30. The results for the wire sizes are shown.
Without checking all of the design conditions, it is obvious that none of the wire sizes satisfy \( n_s \geq 1.2 \). Also, the Gerber line is closer to the yield line than the Goodman. Setting \( n_f = 1.5 \) for Goodman makes it impossible to reach the yield line \( (n_s < 1) \). The table below uses \( n_f = 2 \).

<table>
<thead>
<tr>
<th>Iteration of ( d ) for the first trial</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d )</td>
</tr>
<tr>
<td>( m )</td>
</tr>
<tr>
<td>( A )</td>
</tr>
<tr>
<td>( S_{sw} )</td>
</tr>
<tr>
<td>( S_{sy} )</td>
</tr>
<tr>
<td>( S_{sx} )</td>
</tr>
<tr>
<td>( S_{sa} )</td>
</tr>
<tr>
<td>( \alpha )</td>
</tr>
<tr>
<td>( \beta )</td>
</tr>
<tr>
<td>( C )</td>
</tr>
<tr>
<td>( D )</td>
</tr>
</tbody>
</table>

The satisfactory spring has design specifications of: A313 stainless wire, unpeened, squared and ground, \( d = 0.0915 \) in, OD = 0.811 + 0.092 = 0.903 in, \( L_0 = 4.640 \) in, and \( \mathcal{N}_f = 19.3 \) turns.  \textit{Ans.}

\[ 10-32 \]  This is the same as Prob. 10-30 since \( S_{sw} = 35 \) kpsi. Therefore, the specifications are: A313 stainless wire, unpeened, squared and ground, \( d = 0.0915 \) in, OD = 0.879 + 0.092 =
0.971 in, \( L_0 = 3.606 \) in, and \( N_t = 15.59 \) turns \hspace{1cm} \text{Ans.}

**10-33** For the Gerber-Zimmerli fatigue-failure criterion, \( S_{sa} = 0.67S_{su} \),

\[
S_{se} = \frac{S_{su}}{1 - (S_{su} / S_{sa})^2}, \quad S_{sa} = \frac{r^2S_{se}^2}{2S_{se}} \left[ -1 + \sqrt{1 + \left( \frac{2S_{se}}{rS_{sa}} \right)^2} \right]
\]

The equation for \( S_{sa} \) is the basic difference. The last 2 columns of diameters of Ex. 10-5 are presented below with additional calculations.

<table>
<thead>
<tr>
<th>( d )</th>
<th>0.105</th>
<th>0.112</th>
<th>( d )</th>
<th>0.105</th>
<th>0.112</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_{st} )</td>
<td>278.691</td>
<td>276.096</td>
<td>( N_t )</td>
<td>8.915</td>
<td>6.190</td>
</tr>
<tr>
<td>( S_{su} )</td>
<td>186.723</td>
<td>184.984</td>
<td>( L_0 )</td>
<td>1.146</td>
<td>0.917</td>
</tr>
<tr>
<td>( S_{se} )</td>
<td>38.325</td>
<td>38.394</td>
<td>( L_0 )</td>
<td>3.446</td>
<td>3.217</td>
</tr>
<tr>
<td>( S_{sy} )</td>
<td>125.411</td>
<td>124.243</td>
<td>((L_0)_{cr} )</td>
<td>6.630</td>
<td>8.160</td>
</tr>
<tr>
<td>( S_{sa} )</td>
<td>34.658</td>
<td>34.652</td>
<td>( K_B )</td>
<td>1.111</td>
<td>1.095</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>23.105</td>
<td>23.101</td>
<td>( \tau_a )</td>
<td>23.105</td>
<td>23.101</td>
</tr>
<tr>
<td>( \beta )</td>
<td>1.732</td>
<td>1.523</td>
<td>( n_f )</td>
<td>1.500</td>
<td>1.500</td>
</tr>
<tr>
<td>( C )</td>
<td>12.004</td>
<td>13.851</td>
<td>( \tau )</td>
<td>70.855</td>
<td>70.844</td>
</tr>
<tr>
<td>( D )</td>
<td>1.260</td>
<td>1.551</td>
<td>( n_s )</td>
<td>1.770</td>
<td>1.754</td>
</tr>
<tr>
<td>( ID )</td>
<td>1.155</td>
<td>1.439</td>
<td>( f_n )</td>
<td>105.433</td>
<td>106.922</td>
</tr>
<tr>
<td>( OD )</td>
<td>1.365</td>
<td>1.663</td>
<td>( f_{om} )</td>
<td>-0.973</td>
<td>-1.022</td>
</tr>
</tbody>
</table>

There are only slight changes in the results.

**10-34** As in Prob. 10-34, the basic change is \( S_{sa} \).

For Goodman,

\[
S_{se} = \frac{S_{su}}{1 - (S_{su} / S_{sa})^2}
\]

Recalculate \( S_{sa} \) with

\[
S_{sa} = \frac{rS_{se}S_{su}}{rS_{su} + S_{se}}
\]

Calculations for the last 2 diameters of Ex. 10-5 are given below.
There are only slight differences in the results.

10-35 Use: \( E = 28.6 \text{ Mpsi}, G = 11.5 \text{ Mpsi}, A = 140 \text{ kpsi} \cdot \text{in}^m, m = 0.190, \text{rel cost} = 1 \).

Try \( d = 0.067 \text{ in}, \) \( S_{ut} = \frac{140}{(0.067)^{0.190}} = 234.0 \text{ kpsi} \)

Table 10-6: \( S_y = 0.45S_{ut} = 105.3 \text{ kpsi} \)
Table 10-7: \( S_y = 0.75S_{ut} = 175.5 \text{ kpsi} \)

Eq. (10-34) with \( D/d = C \) and \( C_1 = C \)

\[
\sigma = \frac{F_{max}}{\pi d^2}[(K)_A(16C) + 4] = \frac{S_y}{n_y}
\]

\[
4C^2 - C - 1 = \frac{16C}{4C(C-1)} + 4 = \frac{\pi d^2 S_y}{n_y F_{max}}
\]

\[
4C^2 - C - 1 = (C-1) \left( \frac{\pi d^2 S_y}{4n_y F_{max}} - 1 \right)
\]

\[
C^2 - \frac{1}{4} \left[ 1 + \frac{\pi d^2 S_y}{4n_y F_{max}} - 1 \right] C + \frac{1}{4} \left( \frac{\pi d^2 S_y}{4n_y F_{max}} - 2 \right) = 0
\]

\[
C = \frac{1}{2} \left[ \frac{\pi d^2 S_y}{16n_y F_{max}} \pm \sqrt{\left( \frac{\pi d^2 S_y}{16n_y F_{max}} \right)^2 - \frac{\pi d^2 S_y}{4n_y F_{max}} + 2} \right] \text{ take positive root}
\]

\[
= \frac{1}{2} \left[ \frac{\pi(0.067^2)(175.5)(10^7)}{16(1.5)(18)} + \sqrt{\left( \frac{\pi(0.067^2)(175.5)(10^7)}{16(1.5)(18)} \right)^2 - \frac{\pi(0.067^2)(175.5)(10^7)}{4(1.5)(18)} + 2} \right] = 4.590
\]
\[ D = Cd = 4.59(0.067) = 0.3075 \text{ in} \]
\[ F_i = \frac{\pi d^3}{8D} = \frac{\pi d^3}{8D} \left[ \frac{33.500}{\exp(0.105C)} \pm 1000 \left( 4 - \frac{C - 3}{6.5} \right) \right] \]

Use the lowest \( F_i \) in the preferred range. This results in the best form.

\[ F_i = \frac{\pi(0.067)^3}{8(0.3075)} \left[ \frac{33.500}{\exp(0.105(4.590))} - 1000 \left( 4 - \frac{4.590 - 3}{6.5} \right) \right] = 6.505 \text{ lbf} \]

For simplicity, we will round up to the next integer or half integer. Therefore, use \( F_i = 7 \) lbf

\[ k = \frac{18 - 7}{0.5} = 22 \text{ lbf/in} \]

\[ N_a = \frac{d^4G}{8kD^3} = \frac{(0.067)^4(11.5)(10^6)}{8(22)(0.3075)^3} = 45.28 \text{ turns} \]

\[ N_b = \frac{G}{E} = \frac{45.28 - \frac{11.5}{28.6}}{28.6} = 44.88 \text{ turns} \]

\[ L_0 = (2C - 1 + N_b)d = (2(4.590) - 1 + 44.88)(0.067) = 3.555 \text{ in} \]

\[ L_{bf} = 3.555 + 0.5 = 4.055 \text{ in} \]

**Body:**

\[ K_B = \frac{4C + 2}{4C - 3} = \frac{4(4.590) + 2}{4(4.590) - 3} = 1.326 \]

\[ \tau_{\text{max}} = \frac{8K_B F_{\text{max}}D}{\pi d^3} = \frac{8(1.326)(18)(0.3075)}{\pi(0.067)^3}(10^{-3}) = 62.1 \text{ kpsi} \]

\[ (n_y)_{\text{body}} = \frac{S_{\text{av}}}{\tau_{\text{max}}} = \frac{105.3}{62.1} = 1.70 \]

\[ r_2 = 2d = 2(0.067) = 0.134 \text{ in}, \quad C_2 = \frac{2r_2}{d} = \frac{2(0.134)}{0.067} = 4 \]

\[ (K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4) - 1}{4(4) - 4} = 1.25 \]

\[ \tau_B = (K)_B \left[ \frac{8F_{\text{max}}D}{\pi d^3} \right] = 1.25 \left[ \frac{8(18)(0.3075)}{\pi(0.067)^3} \right](10^{-3}) = 58.58 \text{ kpsi} \]

\[ (n_y)_B = \frac{S_{\text{av}}}{\tau_B} = \frac{105.3}{58.58} = 1.80 \]

\[ \text{fom} = -(1) \frac{\pi d^2(N_k + 2)D}{4} = -\pi^2(0.067)^2(44.88 + 2)(0.3075) = -0.160 \]

Several diameters, evaluated using a spreadsheet, are shown below.
Except for the 0.067 in wire, all springs satisfy the requirements of length and number of coils. The 0.085 in wire has the highest fom.

10-36 Given: \(N_b = 84\) coils, \(F_i = 16\) lbf, OQ&T steel, OD = 1.5 in, \(d = 0.162\) in.

\(D = OD - d = 1.5 - 0.162 = 1.338\) in

(a) Eq. (10-39):
\[
L_0 = 2(D - d) + (N_b + 1)d = 2(1.338 - 0.162) + (84 + 1)(0.162) = 16.12\text{ in} \quad \text{Ans.}
\]
or
\[
\frac{2d + L_0}{2} = 2(0.162) + 16.12 = 16.45\text{ in overall}
\]

(b)
\[
C = \frac{D}{d} = \frac{1.338}{0.162} = 8.26
\]
\[
K_B = \frac{4C + 2}{4C - 3} = \frac{4(8.26) + 2}{4(8.26) - 3} = 1.166
\]
\[
\tau_i = K_B \left[ \frac{8F_i D^3}{\pi d^6} \right] = 1.166 \frac{8(16)(1.338)^3}{\pi(0.162)^6} = 14950\text{ psi} \quad \text{Ans.}
\]

(c) From Table 10-5 use: \(G = 11.4(10^6)\) psi and \(E = 28.5(10^6)\) psi

\[
N_a = N_b + \frac{G}{E} = 84 + \frac{11.4}{28.5} = 84.4 \text{ turns}
\]
\[
k = \frac{d^4G}{8D^3N_a} = \frac{(0.162)^4(11.4)(10^6)}{8(1.338)^3(84.4)} = 4.855\text{ lbf/in} \quad \text{Ans.}
\]
(d) Table 10-4:  \[ A = 147 \text{ psi} \cdot \text{in}^m, \quad m = 0.187 \]

\[ S_{st} = \frac{147}{(0.162)^{0.187}} = 207.1 \text{ kpsi} \]

\[ S_y = 0.75(207.1) = 155.3 \text{ kpsi} \]

\[ S_{sy} = 0.50(207.1) = 103.5 \text{ kpsi} \]

**Body**

\[ F = \frac{\pi d^3 S_{st}}{\pi K_y D} \]

\[ = \frac{\pi(0.162)^3(103.5)(10^3)}{8(1.166)(1.338)} = 110.8 \text{ lbf} \]

**Torsional stress on hook point B**

\[ C_2 = \frac{2r_2}{d} = \frac{2(0.25 + 0.162 / 2)}{0.162} = 4.086 \]

\[ (K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4.086) - 1}{4(4.086) - 4} = 1.243 \]

\[ F = \frac{\pi(0.162)^3(103.5)(10^3)}{8(1.243)(1.338)} = 103.9 \text{ lbf} \]

**Normal stress on hook point A**

\[ C_1 = \frac{2r_1}{d} = \frac{1.338}{0.162} = 8.26 \]

\[ (K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} = \frac{4(8.26)^2 - 8.26 - 1}{4(8.26)(8.26 - 1)} = 1.099 \]

\[ S_{st} = \sigma = F \left[ \frac{16(K)_A D}{\pi d^3} + \frac{4}{\pi d^2} \right] \]

\[ F = \frac{155.3(10^3)}{[16(1.099)(1.338)] / \left[ \frac{\pi(0.162)^3}{2} \right] + \left\{ 4 / \left[ \frac{\pi(0.162)^2}{2} \right] \right\} = 85.8 \text{ lbf} \]

\[ = \min(110.8, 103.9, 85.8) = 85.8 \text{ lbf} \quad \text{Ans.} \]

(e) Eq. (10-48):

\[ y = \frac{F - F_i}{k} = \frac{85.8 - 16}{4.855} = 14.4 \text{ in} \quad \text{Ans.} \]

---

10-37 \[ F_{\min} = 9 \text{ lbf}, \quad F_{\max} = 18 \text{ lbf} \]

\[ F_u = \frac{18 - 9}{2} = 4.5 \text{ lbf}, \quad F_w = \frac{18 + 9}{2} = 13.5 \text{ lbf} \]
A313 stainless: 

\[ 0.013 \leq d \leq 0.1 \quad A = 169 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.146 \]

\[ 0.1 \leq d \leq 0.2 \quad A = 128 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.263 \]

\[ E = 28 \text{ Mpsi}, \quad G = 10 \text{ Gpsi} \]

Try \( d = 0.081 \) in and refer to the discussion following Ex. 10-7

\[ S_u = \frac{169}{(0.081)^{0.146}} = 243.9 \text{ kpsi} \]

\[ S_{sa} = 0.67S_u = 163.4 \text{ kpsi} \]

\[ S_{sy} = 0.35S_u = 85.4 \text{ kpsi} \]

\[ S_y = 0.55S_u = 134.2 \text{ kpsi} \]

Table 10-8:

\[ S_r = 0.45S_u = 109.8 \text{ kpsi} \]

\[ S_v = \frac{S_r}{2} / \left(1 - \left[\frac{S_r}{(2S_v)}\right]^{2}\right) = \frac{109.8}{2} / \left(1 - \left[\frac{109.8}{2} / 243.9\right]^{2}\right) = 57.8 \text{ kpsi} \]

\[ r = F_u / F_m = 4.5 / 13.5 = 0.333 \]

Table 6-7:

\[ S_u = \frac{r^2S_u^2}{2S_v} \left[ -1 + \sqrt{1 + \left(\frac{2S_v}{rS_u}\right)^2} \right] \]

\[ S_u = \frac{(0.333)^2(243.9)^2}{2(57.8)} \left[ -1 + \sqrt{1 + \left(\frac{2(57.8)}{0.333(243.9)}\right)^2} \right] = 42.2 \text{ kpsi} \]

Hook bending

\[ (\sigma_r)_A = F_u \left[ (K)_{\frac{16C}{\pi d^2}} + \frac{4}{\pi d^2} \right] = \frac{S_u}{(n_f)_{\frac{16C}{4C(C-1)}} + 4} = \frac{S_u}{2} \]

This equation reduces to a quadratic in \( C \) (see Prob. 10-35). The useable root for \( C \) is

\[
C = 0.5 \left[ \frac{\pi d^2S_u}{144} + \sqrt{\left(\frac{\pi d^2S_u}{144}\right)^2 - \frac{\pi d^2S_u}{36} + 2} \right]
\]

\[
= 0.5 \left[ \frac{\pi(0.081)^2(42.2)(10^1)}{144} + \sqrt{\left(\frac{\pi(0.081)^2(42.2)(10^1)}{144}\right)^2 - \frac{\pi(0.081)^2(42.2)(10^1)}{36} + 2} \right]
\]

\[ = 4.91 \]
\[
D = Cd = 0.398 \text{ in}
\]
\[
F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D} \left[ \frac{33500}{\exp(0.105C)} \pm 1000 \left(4 - \frac{C - 3}{6.5}\right) \right]
\]

Use the lowest \( F_i \) in the preferred range.
\[
F_i = \frac{\pi (0.081)^3}{8(0.398)} \left[ \frac{33500}{\exp[0.105(4.91)]} - 1000 \left(4 - \frac{4.91 - 3}{6.5}\right) \right]
\]
\[
= 8.55 \text{ lbf}
\]

For simplicity we will round up to next 1⁄4 integer.
\[
F_i = 8.75 \text{ lbf}
\]
\[
k = \frac{18 - 9}{0.25} = 36 \text{ lbf/in}
\]
\[
N_a = \frac{d^4G}{8kD^3} = \frac{(0.081)^4(10)(10^6)}{8(36)(0.398)^3} = 23.7 \text{ turns}
\]
\[
N_b = \frac{N_a - G}{E} = \frac{23.7 - 10}{28} = 23.3 \text{ turns}
\]
\[
L_0 = (2C - 1 + N_b)d = [2(4.91) - 1 + 23.3](0.081) = 2.602 \text{ in}
\]
\[
L_{\text{max}} = L_0 + (F_{\text{max}} - F_i) / k = 2.602 + (18 - 8.75) / 36 = 2.859 \text{ in}
\]
\[
(\sigma_a)_A = \frac{4.5(4)}{\pi d^2} \left(\frac{4C^2 - C - 1}{C - 1} + 1\right)
\]
\[
= \frac{18(10^3)}{\pi (0.081)^2} \left[ \frac{4(4.91)^2 - 4.91 - 1}{4.91 - 1} + 1 \right] = 21.1 \text{ kpsi}
\]
\[
(n_f)_A = \frac{S_a}{(\sigma_a)_A} = \frac{42.2}{21.1} = 2 \text{ checks}
\]

Body:
\[
K_B = \frac{4C + 2}{4C - 3} = \frac{4(4.91) + 2}{4(4.91) - 3} = 1.300
\]
\[
\tau_a = \frac{8(1.300)(4.5)(0.398)}{\pi (0.081)^3(10^3)} = 11.16 \text{ kpsi}
\]
\[
\tau_m = \frac{F_m}{F_a} \tau_a = \frac{13.5}{4.5}(11.16) = 33.47 \text{ kpsi}
\]

The repeating allowable stress from Table 10-8 is
\[
S_{sr} = 0.30S_{ut} = 0.30(243.9) = 73.17 \text{ kpsi}
\]

The Gerber intercept is given by Eq. (10-42) as
\[
S_w = \frac{73.17 / 2}{1 - [(73.17 / 2) / 163.4]^2} = 38.5 \text{ kpsi}
\]
From Table 6-7,

\[ (n_f)_{\text{body}} = \frac{1}{2} \left( \frac{163.4}{33.47} \right) \left( \frac{11.16}{38.5} \right) \left\{ -1 + \sqrt{1 + \left[ \frac{2(33.47)10.73}{163.4(11.16)} \right]^2} \right\} = 2.53 \]

Let \( r_2 = 2d = 2(0.081) = 0.162 \)

\[ C_2 = \frac{2r_2}{d} = 4, \quad (K)_B = \frac{4(4) - 1}{4(4) - 4} = 1.25 \]

\[ (\tau_a)_B = \frac{(K)_B}{K_B} \tau_a = \frac{1.25}{1.30} = (11.16) = 10.73 \text{ kpsi} \]

\[ (\tau_m)_B = \frac{(K)_B}{K_B} \tau_m = \frac{1.25}{1.30} (33.47) = 32.18 \text{ kpsi} \]

Table 10-8: \( (S_{sr})_B = 0.28S_{ut} = 0.28(243.9) = 68.3 \text{ kpsi} \)

\[ (S_{sr})_B = \frac{68.3}{2} \left( \frac{1 - [(68.3 / 2) / 163.4]}{1} \right) = 35.7 \text{ kpsi} \]

\[ (n_f)_B = \frac{1}{2} \left( \frac{163.4}{32.18} \right) \left( \frac{10.73}{35.7} \right) \left\{ -1 + \sqrt{1 + \left[ \frac{2(32.18)10.73}{163.4(10.73)} \right]^2} \right\} = 2.51 \]

**Yield**

**Bending:**

\[ (\sigma_y)_{\text{max}} = \frac{4F_{\text{max}}}{\pi d^2} \left[ \frac{(4C^2 - C - 1)}{C - 1} + 1 \right] \]

\[ = \frac{4(18)}{\pi(0.081^2)} \left[ \frac{(4(4.91)^2 - 4.91 - 1)}{4.91 - 1} + 1 \right](10^3) = 84.4 \text{ kpsi} \]

\[ (n_f)_A = \frac{134.2}{84.4} = 1.59 \]

**Body:**

\[ \tau_y = (F_y / F) \tau_a = (8.75 / 4.5)(11.16) = 21.7 \text{ kpsi} \]

\[ r = \tau_y / (\tau_m - \tau_y) = 11.16 / (33.47 - 21.7) = 0.948 \]

\[ (S_{sa})_y = \frac{r}{r + 1} (S_{sy} - \tau_y) = \frac{0.948}{0.948 + 1} (85.4 - 21.7) = 31.0 \text{ kpsi} \]

\[ (n_f)_{\text{body}} = \frac{(S_{sa})_y}{\tau_a} = \frac{31.0}{11.16} = 2.78 \]

**Hook shear:**

\[ S_{sy} = 0.3S_{ut} = 0.3(243.9) = 73.2 \text{ kpsi} \]

\[ \tau_{\text{max}} = (\tau_a)_B + (\tau_m)_B = 10.73 + 32.18 = 42.9 \text{ kpsi} \]

\[ (n_f)_B = \frac{73.2}{42.9} = 1.71 \]

\[ \text{fom} = -\frac{7.6\pi^2 d^2 (N_b)}{4} = -\frac{7.6\pi^2 (0.081^2)(23.3 + 2)(0.398)}{4} = -1.239 \]
A tabulation of several wire sizes follow

<table>
<thead>
<tr>
<th>$d$</th>
<th>0.081</th>
<th>0.085</th>
<th>0.092</th>
<th>0.098</th>
<th>0.105</th>
<th>0.12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{su}$</td>
<td>163.427</td>
<td>162.281</td>
<td>160.416</td>
<td>158.943</td>
<td>157.350</td>
<td>154.312</td>
</tr>
<tr>
<td>$S_r$</td>
<td>109.764</td>
<td>108.994</td>
<td>107.742</td>
<td>106.753</td>
<td>105.683</td>
<td>103.643</td>
</tr>
<tr>
<td>$S_e$</td>
<td>57.809</td>
<td>57.403</td>
<td>56.744</td>
<td>56.223</td>
<td>55.659</td>
<td>54.585</td>
</tr>
<tr>
<td>$S_a$</td>
<td>42.136</td>
<td>41.841</td>
<td>41.360</td>
<td>40.980</td>
<td>40.570</td>
<td>39.786</td>
</tr>
<tr>
<td>$C$</td>
<td>4.903</td>
<td>5.484</td>
<td>6.547</td>
<td>7.510</td>
<td>8.693</td>
<td>11.451</td>
</tr>
<tr>
<td>$D$</td>
<td>0.397</td>
<td>0.466</td>
<td>0.602</td>
<td>0.736</td>
<td>0.913</td>
<td>1.374</td>
</tr>
<tr>
<td>OD</td>
<td>0.478</td>
<td>0.551</td>
<td>0.694</td>
<td>0.834</td>
<td>1.018</td>
<td>1.494</td>
</tr>
<tr>
<td>$F_i$ (calc)</td>
<td>8.572</td>
<td>7.874</td>
<td>6.798</td>
<td>5.987</td>
<td>5.141</td>
<td>3.637</td>
</tr>
<tr>
<td>$F_i$ (rd)</td>
<td>8.75</td>
<td>9.75</td>
<td>10.75</td>
<td>11.75</td>
<td>12.75</td>
<td>13.75</td>
</tr>
<tr>
<td>$k$</td>
<td>36.000</td>
<td>36.000</td>
<td>36.000</td>
<td>36.000</td>
<td>36.000</td>
<td>36.000</td>
</tr>
<tr>
<td>$N_a$</td>
<td>23.86</td>
<td>17.90</td>
<td>11.38</td>
<td>8.03</td>
<td>5.55</td>
<td>2.77</td>
</tr>
<tr>
<td>$N_b$</td>
<td>23.50</td>
<td>17.54</td>
<td>11.02</td>
<td>7.68</td>
<td>5.19</td>
<td>2.42</td>
</tr>
<tr>
<td>$L_0$</td>
<td>2.617</td>
<td>2.338</td>
<td>2.127</td>
<td>2.126</td>
<td>2.266</td>
<td>2.918</td>
</tr>
<tr>
<td>$L_{18}$</td>
<td>2.874</td>
<td>2.567</td>
<td>2.328</td>
<td>2.300</td>
<td>2.412</td>
<td>3.036</td>
</tr>
<tr>
<td>$(\eta)_A$</td>
<td>2.000</td>
<td>2.000</td>
<td>2.000</td>
<td>2.000</td>
<td>2.000</td>
<td>2.000</td>
</tr>
<tr>
<td>$K_B$</td>
<td>1.301</td>
<td>1.264</td>
<td>1.216</td>
<td>1.185</td>
<td>1.157</td>
<td>1.117</td>
</tr>
<tr>
<td>$(\tau)_B$</td>
<td>11.141</td>
<td>10.994</td>
<td>10.775</td>
<td>10.617</td>
<td>10.457</td>
<td>10.177</td>
</tr>
<tr>
<td>$(\sigma)_B$</td>
<td>33.424</td>
<td>32.982</td>
<td>32.326</td>
<td>31.852</td>
<td>31.372</td>
<td>30.532</td>
</tr>
<tr>
<td>$(\eta)_B$</td>
<td>73.176</td>
<td>72.663</td>
<td>71.828</td>
<td>71.169</td>
<td>70.455</td>
<td>69.095</td>
</tr>
<tr>
<td>$(S_e)$</td>
<td>38.519</td>
<td>38.249</td>
<td>37.809</td>
<td>37.462</td>
<td>37.087</td>
<td>36.371</td>
</tr>
<tr>
<td>$(K)_B$</td>
<td>2.531</td>
<td>2.547</td>
<td>2.569</td>
<td>2.583</td>
<td>2.596</td>
<td>2.616</td>
</tr>
<tr>
<td>$(\tau)_B$</td>
<td>1.250</td>
<td>1.250</td>
<td>1.250</td>
<td>1.250</td>
<td>1.250</td>
<td>1.250</td>
</tr>
<tr>
<td>$S_y$</td>
<td>134.156</td>
<td>133.215</td>
<td>131.685</td>
<td>130.476</td>
<td>129.168</td>
<td>126.674</td>
</tr>
<tr>
<td>$(\sigma)_A$</td>
<td>84.273</td>
<td>83.682</td>
<td>82.720</td>
<td>81.961</td>
<td>81.139</td>
<td>79.573</td>
</tr>
<tr>
<td>$(\eta)_A$</td>
<td>1.592</td>
<td>1.592</td>
<td>1.592</td>
<td>1.592</td>
<td>1.592</td>
<td>1.592</td>
</tr>
<tr>
<td>$\tau$</td>
<td>21.663</td>
<td>23.820</td>
<td>25.741</td>
<td>27.723</td>
<td>29.629</td>
<td>31.097</td>
</tr>
<tr>
<td>$r$</td>
<td>0.945</td>
<td>1.157</td>
<td>1.444</td>
<td>1.942</td>
<td>2.906</td>
<td>4.703</td>
</tr>
<tr>
<td>$(S_y)_B$</td>
<td>85.372</td>
<td>84.773</td>
<td>83.800</td>
<td>83.030</td>
<td>82.198</td>
<td>80.611</td>
</tr>
<tr>
<td>$(\sigma)_B$</td>
<td>30.958</td>
<td>32.688</td>
<td>34.302</td>
<td>36.507</td>
<td>39.109</td>
<td>40.832</td>
</tr>
<tr>
<td>$(S_y)_B$</td>
<td>73.176</td>
<td>72.663</td>
<td>71.828</td>
<td>71.169</td>
<td>70.455</td>
<td>69.095</td>
</tr>
<tr>
<td>$(\tau)_B$</td>
<td>42.819</td>
<td>43.486</td>
<td>44.321</td>
<td>44.801</td>
<td>45.177</td>
<td>45.564</td>
</tr>
<tr>
<td>$(\eta)_B$</td>
<td>1.709</td>
<td>1.671</td>
<td>1.621</td>
<td>1.589</td>
<td>1.560</td>
<td>1.516</td>
</tr>
</tbody>
</table>

form

| $d$ | 0.1246 | 0.1234 | 0.1245 | 0.1283 | 0.1357 | 0.1639 |

optimal form
The shaded areas show the conditions not satisfied.

10-38 For the hook,

\[ M = FR \sin \theta, \quad \frac{\partial M}{\partial F} = R \sin \theta \]

\[ \delta_f = \frac{1}{EI} \int_{\theta_0}^{\pi/2} F (R \sin \theta)^2 R d\theta = \frac{\pi FR^3}{2EI} \]

The total deflection of the body and the two hooks

\[ \delta = \frac{8FD^3N_b}{d^4G} + 2 \left( \frac{\pi FR^3}{2EI} \right) = \frac{8FD^3N_b}{d^4G} + \frac{\pi F(D/2)^3}{E(\pi/64)(d^4)} \]

\[ = \frac{8FD^3}{d^4G} \left( N_b + \frac{G}{E} \right) = \frac{8FD^3N_u}{d^4G} \]

\[ \therefore N_u = N_b + \frac{G}{E} \quad \text{Q.E.D.} \]

10-39 Table 10-5 \((d = 4 \text{ mm} = 0.1575 \text{ in}): \ E = 196.5 \text{ GPa} \)

<table>
<thead>
<tr>
<th>Table 10-4 for A227:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = 1783 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.190 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Eq. (10-14):</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_{ut} = \frac{A}{d^m} = \frac{1783}{4^{0.190}} = 1370 \text{ MPa} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Eq. (10-57):</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_y = \sigma_{all} = 0.78 \cdot S_{ut} = 0.78(1370) = 1069 \text{ MPa} )</td>
</tr>
</tbody>
</table>

\( D = OD - d = 32 - 4 = 28 \text{ mm} \)

\( C = D/d = 28/4 = 7 \)

<table>
<thead>
<tr>
<th>Eq. (10-43):</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_i = \frac{4C^2 - C - 1}{4C(C - 1)} = \frac{4(7^2) - 7 - 1}{4(7)(7 - 1)} = 1.119 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Eq. (10-44):</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma = K_i \frac{32Fr}{\pi d^4} )</td>
</tr>
</tbody>
</table>

At yield, \( Fr = M_y, \ \sigma = S_y \). Thus,

\[ M_y = \frac{\pi d^3 S_y}{32K_i} = \frac{\pi (4^3)1069(10^{-3})}{32(1.119)} = 6.00 \text{ N} \cdot \text{m} \]
Count the turns when $M = 0$

$$N = 2.5 - \frac{M_y}{k}$$

where from Eq. (10-51):

$$k = \frac{d^4 E}{10.8DN}$$

Thus,

$$N = 2.5 - \frac{M_y}{d^4 E / (10.8DN)}$$

Solving for $N$ gives

$$N = \frac{2.5}{1 + [10.8DM_y / (d^4 E)]}$$

$$= \frac{2.5}{1 + \left[\frac{10.8(28)(6.00)}{4^4(196.5)}\right]} = 2.413 \text{ turns}$$

This means $(2.5 - 2.413)(360^\circ)$ or $31.3^\circ$ from closed. \textit{Ans.}

Treating the hand force as in the middle of the grip,

$$r = 112.5 - 87.5 + \frac{87.5}{2} = 68.75 \text{ mm}$$

$$F_{max} = \frac{M_y}{r} = \frac{6.00(10^3)}{68.75} = 87.3 \text{ N} \quad \text{Ans.}$$

\textbf{10-40} The spring material and condition are unknown. Given $d = 0.081 \text{ in}$ and OD = 0.500,

(a) $D = 0.500 - 0.081 = 0.419 \text{ in}$

Using $E = 28.6 \text{ Mpsi}$ for an estimate

$$k' = \frac{d^4 E}{10.8DN} = \frac{(0.081)^4(28.6)(10^6)}{10.8(0.419)(11)} = 24.7 \text{ lbf} \cdot \text{ in/turn}$$

for each spring. The moment corresponding to a force of 8 lbf

$$Fr = (8/2)(3.3125) = 13.25 \text{ lbf} \cdot \text{ in/spring}$$

The fraction windup turn is

$$n = \frac{Fr}{k'} = \frac{13.25}{24.7} = 0.536 \text{ turns}$$

The arm swings through an arc of slightly less than $180^\circ$, say $165^\circ$. This uses up $165/360$ or 0.458 turns. So $n = 0.536 - 0.458 = 0.078$ turns are left (or
0.078(360°) = 28.1°. The original configuration of the spring was

![Spring Diagram](image)

**Ans.**

(b)

\[
C = \frac{D}{d} = \frac{0.419}{0.081} = 5.17
\]

\[
K_i = \frac{4C^2 - C - 1}{4C(C - 1)} = \frac{4(5.17)^2 - 5.17 - 1}{4(5.17)(5.17 - 1)} = 1.168
\]

\[
\sigma = K_i \frac{32M}{\pi d^3} = 1.168 \left[ \frac{32(13.25)}{\pi (0.081)^3} \right] = 297 (10^3) \text{ psi} = 297 \text{ kpsi} \quad \text{Ans.}
\]

To achieve this stress level, the spring had to have set removed.

---

10-41 (a) Consider half and double results

**Straight section:**

\[ M = 3FR, \quad \frac{\partial M}{\partial F} = 3R \]

**Upper 180° section:**

\[ M = F[R + R(1 - \cos \phi)] = FR(2 - \cos \phi), \quad \frac{\partial M}{\partial F} = R(2 - \cos \phi) \]

**Lower section:**

\[ M = FR \sin \theta, \quad \frac{\partial M}{\partial F} = R \sin \theta \]

Considering bending only:
\[ \delta = \frac{\partial U}{\partial F} = 2 \int_{0}^{1/2} 9FR^2 \, dx + \int_{0}^{\pi} FR^2(2 - \cos \phi)^2 R \, d\phi + \int_{0}^{\pi/2} F(R \sin \theta)^2 R \, d\theta \]
\[ = \frac{2F}{EI} \left[ \frac{9}{2} R^2 l + R^3 \left( 4\pi - 4\sin \phi_{0} + \frac{\pi}{2} \right) + R^3 \left( \frac{\pi}{4} \right) \right] \]
\[ = \frac{2FR^2}{EI} \left( \frac{19\pi}{4} R + \frac{9}{2} l \right) = \frac{FR^2}{2EI}(19\pi R + 18l) \]

The spring rate is
\[ k = \frac{F}{\delta} = \frac{2EI}{R^3(19\pi R + 18l)} \quad \text{Ans.} \]

(b) Given: A227 HD wire, \( d = 2 \text{ mm} \), \( R = 6 \text{ mm} \), and \( l = 25 \text{ mm} \).

Table 10-5 (\( d = 2 \text{ mm} = 0.0787 \text{ in} \)): \( E = 197.2 \text{ GPa} \)

\[ k = \frac{2(197.2)10^9 \pi (0.002^4)}{0.006^4 [19\pi (0.006) + 18(0.025)]} = 10.65 \left(10^3 \right) \text{ N/m} \quad \text{Ans.} \]

(c) The maximum stress will occur at the bottom of the top hook where the bending-moment is \( 3FR \) and the axial force is \( F \). Using curved beam theory for bending.

Eq. (3-65), p. 133: \[ \sigma_i = \frac{Mc_i}{Aer_i} = \frac{3FRc_i}{(\pi d^2 / 4)e(\pi d / 2)} \]

Axial: \[ \sigma_a = \frac{F}{A} = \frac{F}{\pi d^2 / 4} \]

Combining, \[ \sigma_{\text{max}} = \sigma_i + \sigma_a = \frac{4F}{\pi d^2} \left[ \frac{3Rc_i}{e(\pi d / 2)} + 1 \right] = \sigma_y \]

\[ F = \frac{\pi d^2 \sigma_y}{4 \left[ \frac{3Rc_i}{e(\pi d / 2)} + 1 \right]} \quad \text{(1) Ans.} \]

For the clip in part (b),

Eq. (10-14) and Table 10-4: \( S_{ut} = A/d^m = 1783/2^{0.190} = 1563 \text{ MPa} \)

Eq. (10-57): \( S_y = 0.78 \) \( S_{ut} = 0.78(1563) = 1219 \text{ MPa} \)
Table 3-4, p. 135:

\[ r_p = \frac{t^2}{2\left(6 - \sqrt{6^2 - t^2}\right)} = 5.95804 \text{ mm} \]

\[ e = r_c - r_n = 6 - 5.95804 = 0.04196 \text{ mm} \]

\[ c_i = r_n - (R - d/2) = 5.95804 - (6 - 2/2) = 0.95804 \text{ mm} \]

Eq. (1):

\[ F = \pi \left(0.002^2\right)1219 \left(10^6\right) = 46.0 \text{ N} \quad \text{Ans.} \]

\[ \frac{3(6)0.95804}{0.04196(6-1)} + 1 \]

10-42 (a)

\[ M = -Fx, \quad \frac{\partial M}{\partial F} = -x \quad 0 \leq x \leq l \]

\[ M = Fl + FR\left(1 - \cos \theta\right), \quad \frac{\partial M}{\partial F} = l + R\left(1 - \cos \theta\right) \quad 0 \leq \theta \leq \pi / 2 \]

\[ \delta_f = \frac{1}{EI} \left\{ \left[\int_0^l Fx(-x)dx + \int_{\pi/2}^\pi R^2 \left[l + R\left(1 - \cos \theta\right)\right]^2 \, d\theta\right] R\delta \right\} \]

\[ = \frac{F}{12EI} \left\{ 4l^3 + 3R\left[2\pi l^2 + 4\left(\pi^2 - 4\right)lR + \left(3\pi^2 - 8\right)R^2\right] \right\} \]

The spring rate is

\[ k = \frac{F}{\delta_f} = \frac{12EI}{4l^3 + 3R\left[2\pi l^2 + 4\left(\pi^2 - 4\right)lR + \left(3\pi^2 - 8\right)R^2\right]} \quad \text{Ans.} \]

(b) Given: A313 stainless wire, \( d = 0.063 \text{ in} \), \( R = 0.625 \text{ in} \), and \( l = 0.5 \text{ in} \).

Table 10-5: \( E = 28 \text{ Mpsi} \)
\[ I = \frac{\pi}{64} d^4 = \frac{\pi}{64} (0.063^4) = 7.733 \times 10^{-7} \text{ in}^4 \]

\[ k = \frac{12 (28) 10^6 (7.733) 10^{-7}}{4 (0.5^4) + 3 (0.625) [2\pi (0.5^2) + 4 (\pi - 2) 0.5 (0.625) + (3\pi - 8) (0.625^2)]} \]

\[ = 36.3 \text{ lbf/in} \quad \text{Ans.} \]

(c) Table 10-4: \( A = 169 \text{ kpsi-in}^m, \ m = 0.146 \)

Eq. (10-14): \( S_{ut} = A/d^m = 169/0.063^{0.146} = 253.0 \text{ kpsi} \)

Eq. (10-57): \( S_y = 0.61 S_{ut} = 0.61(253.0) = 154.4 \text{ kpsi} \)

One can use curved beam theory as in the solution for Prob. 10-41. However, the equations developed in Sec. 10-12 are equally valid.

\[ C = D/d = 2(0.625 + 0.063/2)/0.063 = 20.8 \]

Eq. (10-43): \( K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4(20.8^2) - 20.8 - 1}{4(20.8)(20.8 - 1)} = 1.037 \)

Eq. (10-44), setting \( \sigma = S_y \):

\[ K_i \frac{32Fr}{\pi d^3} = S_y \quad \Rightarrow \quad 1.037 \frac{32F(0.5 + 0.625)}{\pi (0.063^3)} = 154.4 \times 10^3 \]

Solving for \( F \) yields \( F = 3.25 \text{ lbf} \quad \text{Ans.} \)

Try solving part (c) of this problem using curved beam theory. You should obtain the same answer.

10-43 (a) \( M = -Fx \)

\[ \sigma = \frac{M}{I/c} = \frac{Fx}{I/c} = \frac{Fx}{bh^3/6} \]

Constant stress,

\[ \frac{bh^3}{6} = \frac{Fx}{\sigma} \quad \Rightarrow \quad h = \sqrt[3]{\frac{6Fx}{b\sigma}} \quad (1) \quad \text{Ans.} \]
At \( x = l \),
\[
h_o = \sqrt[3]{\frac{6Fl}{b\sigma}} \quad \Rightarrow \quad h = h_o \sqrt{\frac{x}{l}} \quad \text{Ans.}
\]

(b) \( M = -Fx, \quad \partial M / \partial F = -x \)
\[
y = \int_0^l \frac{M (\partial M / \partial F)}{EI} \, dx = \frac{1}{E} \int_0^l \frac{-Fx(-x)}{\frac{1}{12}bh_o^4 (x/l)^{3/2}} \, dx = \frac{12Fl^{3/2}}{bh_o^4 E} \int_0^l x^{3/2} \, dx
\]
\[
= \frac{2}{3} \frac{12Fl^{3/2}}{bh_o^4 E} = \frac{8Fl^3}{bh_o^4 E}
\]
\[
k = \frac{F}{y} = \frac{bh_o^4 E}{8l^3} \quad \text{Ans.}
\]

**10-44** Computer programs will vary.

**10-45** Computer programs will vary.